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A COMPARISON OF ROWLAND'S MERCURY THERMOMETERS WITH A CALLENDAR-GRIFFITHS' PLATINUM THERMOMETER;

A COMPARISON OF THE PLATINUM THERMOMETER WITH A TONNELOT THERMOMETER STANDARDIZED AT THE BUREAU INTERNATIONAL;

AND A REDUCTION OF ROWLAND'S VALUES OF THE MECHANICAL EQUIVALENT OF HEAT TO THE PARIS NITROGEN SCALE.[1]

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THE recent determinations of the mechanical equivalent of heat by electrical methods, while in fair agreement among themselves, seem to give results considerably higher than those obtained by experimenters using the more direct mechanical methods; these differences, which are probably greater than the errors of experiment, must be due to a difference in the standards of thermometry employed or to a still undiscovered error in the system of electric units. The object of the present investigation was to obtain a connection between Rowland's standard of thermometry and that employed in the electrical determinations of the equivalent, in order to render the different determinations more easily comparable.

[1] An abstract of this investigation was read at the Toronto meeting of the British Association, and a preliminary account appeared in the Johns Hopkins University Circulars, June, 1897, 1898.

Joule compared his thermometer with Rowland's Baudin 6166 (Proc. Am. Acad., 16, p. 38, 1880) and in this way Joule's determinations of the mechanical equivalent have been reduced to the Rowland "air-scale." By a further elaborate comparison by Professor Schuster (Phil. Mag., 39, 1895) of Joule's thermometers with a thermometer standardized at the Bureau International, an indirect connection was obtained between Rowland's air scale and the nitrogen scale of the Bureau International. This comparison pointed to differences in the two scales as great as 0°.05 C. Inasmuch as the details of the method employed by Joule in his comparison are not known, Professor Ames suggested that another more direct comparison of Professor Rowland's thermometers was desirable.

METHOD OF COMPARISON.

As we wished to use the results of this experiment to reduce Rowland's values of the equivalent to the scales used by experimenters employing the electrical methods, and thus render the results comparable, it was deemed best to make the comparisons of these thermometers under conditions as nearly as possible the same as those under which they were employed by him in his experiments on the mechanical equivalent. For this reason a Callendar-Griffiths' platinum thermometer was selected for the comparison. This instrument offers many advantages for the standardization of calorimetric thermometers. The platinum thermometer was subsequently compared with a Tonnelot thermometer standardized at the Bureau International. A connection was thus obtained between Rowland's air scale and the Callendar-Griffiths' air scale as well as with the nitrogen scale of the Bureau International.

The zero readings of the mercury thermometers were taken in the same way that Rowland used them, before each comparison. The thermometers were placed in a metallic vessel well wrapped with thick felt and filled with finely cracked ice of great purity steeped with distilled water. After they had remained in ice about one and one-half hours the zero readings were taken by means of a micrometer telescope. The mercury thermometer was then hastily transferred to a well stirred calorimeter with its bulb near that of the platinum thermometer. The temperature of the water in the calorim-

eter at the start was generally about 2° or 3°C. A series of five or six readings on the mercury thermometer was then taken by one observer with the aid of a micrometer telescope, while the other observer took simultaneous readings of the resistance of the platinum thermometer.

The temperature of the water in the calorimeter was then raised at a rate of about 10° to 15° per hour to the next temperature at which a comparison was to be made; this was accomplished by sending a suitable current through a coil wrapped on the outside of the calorimeter. When the desired temperature was reached, the current was cut off or so regulated that the temperature was very slowly rising, when a further series of simultaneous observations were taken, and so on to the end of the scale.

The mercury thermometers were used in a vertical position, and all observations were taken while the meniscus was very slowly rising, usually at a rate of about .002 or .003° per minute, so that the slight uncertainties of a falling meniscus were avoided. At the point where the thermometer emerges from the calorimeter it is surrounded by a small water jacket in which a small thermometer and stirrer are placed. The temperature of the mercury column above the water jacket is assumed to be at the temperature of the surrounding air as indicated by a thermometer hung within an inch or two of the stem. Thus the stem correction was applied in two parts by means of the formulæ $\Delta N = .000156\, n\, (t - t')$, where ΔN is the correction to the observed stem reading, t the observed temperature, t' the temperature of that portion of the stem emerging from the calorimeter, and n the number of stem divisions at the temperature t'. The stem correction was applied to the observed stem reading and the corresponding temperature on Rowland's air scale was then taken from his Tables (Proc. Am. Acad., 15, 1879, pp. 115 and 116); this temperature was then reduced by the rise of the zero since Rowland's experiments.

The corresponding temperature on the Callendar-Griffiths' air scale was obtained from the observed resistance of the platinum thermometer; the way in which this was done will be explained more fully later.

MANNER IN WHICH BAUDIN THERMOMETERS WERE USED BY PROFESSOR ROWLAND.

These thermometers were first calibrated by measuring the length of a short column of mercury in different portions of the stem, and in this way the relative volumes of different parts of the tube were found. Temperature on the mercury-in-glass thermometers was then defined as proportional to the apparent volume of mercury in glass, when the thermometer is vertical. It was then assumed that the apparent volume of mercury in glass could be expressed as a function of the second degree of the temperature on the air thermometer, of the following form:

$$T = C'V - t_0' - mT(40 - T)\{1 - n(40 + T)\}$$

using the 0° and 40° points as fixed by the air thermometer,

where T is the temperature on the air thermometer,

V is the volume of the stem of the mercurial thermometer, as determined by the calibration, and measured from any arbitrary point,

C, t_0', m, n are constants to be determined.

From a series of direct comparisons of the mercurial thermometers, with a standard air thermometer, a number of equations of the above form were obtained, from which the most probable values of these constants were deduced by the method of least squares.

From these formulæ tables were constructed giving the temperature on the scale of the standard air thermometer corresponding to each centimeter of the stem. These tables are found on pp. 115 and 116, Proc. Am. Acad., 15, 1879.

These thermometers were always used with a constant zero determined before each series of observations, by immersing the thermometer in a mixture of finely cracked ice and distilled water for a period of one or two hours. As the position of the zero, owing to the imperfect elasticity of the glass, depends on the temperature to which the thermometer has been subjected recently, the thermometers were always kept at least one week at a temperature of about

20° C. before they were used in a comparison or in a determination of the mechanical equivalent, in which interval the zero will have almost recovered from the depression due to a recent heating up of the thermometer.

In the original comparisons of these thermometers with the air thermometer, no corrections were necessary for internal pressure due to the mercury column and meniscus, as the thermometers were compared and always used in a vertical position. A correction for external pressure was made, due to about 60 cms. of water in the comparison tank.

In the experiments on the mechanical equivalent the thermometer was placed in the calorimeter with the top of the bulb about 5 cm. below the surface of the water, and the remainder of the stem projected out into the air. The stem correction was then applied in two parts by surrounding the stem, where it projected from the calorimeter, with a small water-jacket, the remainder of the stem being assumed to be at the temperature of the surrounding air.

Platinum Thermometry.

The measurement of temperature by means of the variation of resistance of a wire was first seriously proposed by Siemens who submitted for trial to a committee of the British Association some platinum resistance pyrometers. (B. A. Report, 1874.) The results of those experiments were unfavorable to the use of the platinum resistance thermometer as a scientific instrument of precision. It was found that, for this type of pyrometer, the resistance of the platinum wire was not a constant for a fixed temperature, but depended greatly on the previous history of the wire. The development of this type of thermometer was next taken up by Callendar, who, in an exhaustive series of researches (Phil. Trans. Roy. Soc., Vol. 178, A), showed that if the platinum wire is fairly pure to begin with, and if it has been well annealed, the resistance is always the same at the same temperature, even if the wire has been repeatedly exposed to quite high temperatures in the interval. In the light of these researches it is plain that the failure of the Siemens' resistance pyrometer was simply due to faulty construction, which

exposed the wire to the action of injurious vapors, etc., from the clay on which it was wound and the iron cylinder in which it was enclosed.

The extensive researches of Dewar and Fleming (Phil. Mag., 1893), and Cailletet and Colardeau (Jl. de Phys. T. VIII., 1888), at very low temperatures, and those of Heycock and Neville (Chem. Soc. Trans., 1895) at very high temperatures, taken with those of Callendar and Griffiths (Phil. Trans. Roy. Soc., Vol. 178, A, 1887; Vol. 182, A, 1891), at ordinary temperatures established beyond a doubt the remarkable accuracy and constancy of this instrument over a very wide range of temperature.

Temperature on the platinum scale is defined by the equation

$$pt = \frac{R - R_0}{R_1 - R_0} \times 100,$$

where pt denotes the *platinum temperature*, R the resistance of the platinum coil of the thermometer at that temperature, R_0 its resistance at 0° C., and R_1 the resistance at 100 C. From the above definition it is seen that *platinum temperature* is independent of the unit of resistance employed.

Callendar and Griffiths, in an elaborate comparison of the platinum and air thermometers (Phil. Trans. Roy. Soc., Vol. 182, A, 1891), have shown that the relation between the two scales of temperature can be expressed by an equation of the form

$$t - pt = \delta\{(t/100)^2 - t/100\}$$

where t is the temperature on the air scale, pt the *platinum temperature* as defined above, and δ a constant depending on the specimen of wire used. This relation being a parabolic one, only three temperatures are necessary for the complete standardization of a platinum thermometer, *i. e.*, for the determination of δ. Two temperatures always used for this purpose are that of melting ice, 0°C., and that of water boiling freely under a pressure of 760 mm. of mercury whose density is that of 0°C., sea level, latitude 45°, 100°C. For the third temperature the boiling point of some sub-

stance, which is accurately known from a direct measurement with an air thermometer, is used. The one most frequently used is the boiling point of sulphur, $t = 444°.53$, on account of the accuracy with which it is known. Callendar and Griffiths have shown that if platinum thermometers be standardized by these three temperatures (melting ice, steam, sulphur vapor), the above formula will give temperatures on the air scale to within .01°C., over the range 0°C. to 100°C. This conclusion is further confirmed by a direct comparison made by Dr. W. S. Day and the authors between a platinum thermometer and a Tonnelot thermometer standardized at the Bureau International, whose scale of temperature is based on measurements made with a constant volume gas thermometer. The comparison referred to above, made by Callendar and Griffiths, is based on a constant pressure air thermometer.

Description of Apparatus.

Mercury Thermometers.—The thermometers used by Rowland in his experiments on the mechanical equivalent were constructed by Baudin in 1876–77 and are numbered 6163, 6165 and 6166. Of these thermometers Baudin 6163 is perhaps the most important, as it was used in eight of the fourteen determinations; Baudin 6166 was used in four determinations, and has besides an historic interest, as Joule compared his thermometers with it; Baudin 6165 was used in only one determination of the mechanical equivalent. Besides these, Kew 104, constructed by Welsh in 1853 was used

Thermometer.	Range.	1° C. occupies about.
Baudin 6163	−6° C. to 40° C.	9.0 mm.
" 6165	−3° C. to 33° C.	11.7 "
" 6166	−2° C. to 31° C.	12.9 "

in one determination. This thermometer was not available for this comparison, but inasmuch as it was only calibrated to 0°.5 F. and was given but little weight in the final results, its effect on the values of the mechanical equivalent is entirely negligible.

The stems of the thermometers are graduated in millimeters.

Platinum Thermometer.—The platinum thermometer used in these experiments and shown in Fig. 1 was constructed by the Cambridge Scientific Instrument Company under the supervision of Mr. E. H. Griffiths. It consists of a coil of fine platinum wire wound on a mica frame whose edges are serrated to receive the windings. The thick leads running down to this coil are held apart by mica washers and the whole thermometer is enclosed in a thin glass tube. The thick leads are connected with the terminals marked *PP*. This thermometer was also provided with compensating leads, so that its indications are independent of the depth of stem immersions, pro vided this immersion is sufficient to prevent conduction from the outside along the leads to the coil. The compensating leads *CC* are simply a loop (not connected with the platinum coil) running down the stem, parallel to the main leads *PP* throughout their length, so that they are exposed to the same temperature changes. These compensating leads are adjusted so that their resistance is equal to that of the main leads *PP*. Now, obviously, if these leads *CC* are placed in the other arm of the Wheatstone bridge, as shown in Fig. 2, below, they will exactly compensate for the change in resistance of the leads to the coil, and consequently the resistance we measure, and hence the temperature, is independent of the depth of the immersion of the thermometer in the liquid or vapor whose temperature is sought. A little consideration will show that it is not necessary that the resistance of the compensating leads be exactly equal to the leads to the coil in order to have compensation. It is only necessary that they be so nearly equal that the temperature change of their difference produce no appreciable effect on the resistance.

Calorimeter.—The calorimeter, in which these comparisons were made (shown in Fig. 1), was made of copper and provided with an efficient stirrer which was driven by a small hot-air engine. The outside of the calorimeter was closely wrapped with a coil of single silk-covered German silver wire, so that the temperature of the calorimeter and its contents could be raised at any desired rate by an electric current of suitable strength. The calorimeter was placed inside a large double wall copper vessel (ext. diam. 36 cm., ht. 35

cm.) which surrounded it on all sides except the top. The space between these walls (2.5 cm. apart) could be filled with water at any desired temperature and thus the temperature of the calorimeter could be maintained constant for some minutes or caused to vary at will. The entire arrangement was then surrounded by a large box (46 cm. on the side) and all intervening spaces packed loosely with feathers.

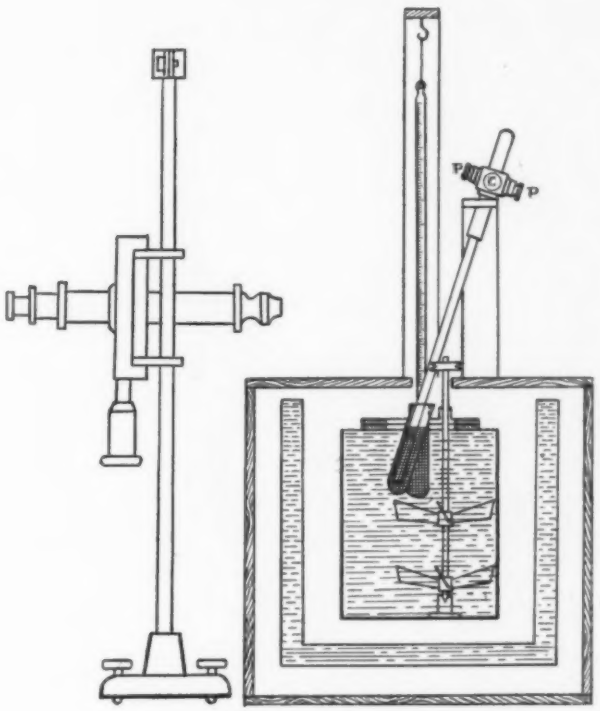

Fig. 1.

Resistance Box.—The resistance measurements of the platinum thermometer were made with a Callendar-Griffiths' resistance box (No. 7) especially designed for the measurement of platinum temperature. The construction and calibration of a resistance box similar to the one used by us has been described by Mr. E. H. Griffiths in Nature, Nov. 14, 1895. A general idea of the con-

struction of the box may be gathered from the following diagrammatic sketch.

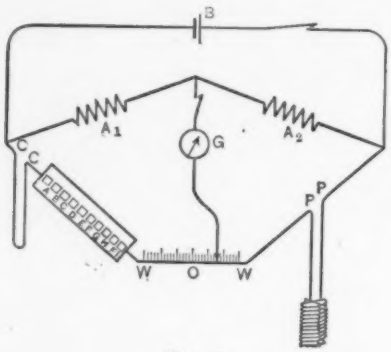

Fig. 2

H	G	F	E	D	C	B	A	FI
5	10	20	40	80	160	320	640	100

A_1, A_2 are two equal coils which serve as the equal arms of a Wheatstone bridge. The coils of the box, which were made of platinum silver, have approximately the following nominal values in box units (approximately, one box unit = 0.01 ohm).

The bridge-wire *WW* was made of platinum silver, was 30 cm. long and had a resistance of about 0.30 ohms. Contact was made with the galvanometer through a similar wire stretched parallel to the bridge-wire. The point of contact was determined by a very carefully constructed contact key carrying a vernier reading directly to 1/50 mm., so that 1/100 mm. was easily estimated (corresponding to about 0.00001 ohm). This ingenious contact key, which was designed by Mr. Horace Darwin, renders any injury to the bridge-wire almost impossible. The scale and bridge-wire are connected in such a way that the readings are practically unaffected by very considerable changes in temperature. The leads to the coil of the platinum thermometer were connected to the terminals *PP* and the compensating leads to the terminals *CC*. The galvanometer is shown at *G*, and the battery at *B*.

When the intervals *C* to *C* and *P* to *P*, were short-circuited by strips of copper, and all plugs inserted, the bridge was balanced when contact was made with the bridge-wire at some point very near the center, *O*. The reading of the bridge-wire when the bridge was balanced under these conditions is called the "zero correction"

to the bridge-wire. The platinum thermometer was then inserted and its resistance at any temperature could be measured by suitable combinations of coils and bridge-wire. The corrections to reduce the nominal values of the coils to mean box units at some standard temperature, as determined by a previous calibration of the box, were then applied, as well as the corrections to reduce the bridge-wire readings to mean box units. The resistance of the platinum thermometer was then known in terms of the mean box unit. Any resistance could be assured with several different combinations of coils and bridge-wire, and thus the accuracy of the measurements was exposed to a severe test.

The top of the box was constructed of white marble, which was found to have a very high insulation resistance. The brass plugs were large and carefully ground, but at no point were they larger than the top of the hole into which they were inserted. This is a very important detail as it insures the impossibility of wearing a shoulder on the plug, in which case no amount of pressure will give good contact.

This box was also provided with small switches by which resistances could be thrown into the battery circuit and the galvanometer shunted, which proved a great convenience in finding an approximate balance.

The bridge was always used with a key by which the circuits could be closed and opened in such a way that the effects of thermo-electric currents were eliminated.

In these experiments the resistance box was supported in a large double-wall copper tank. The inner compartment of this tank, in which the resistance coils are placed, is filled with a highly insulating neutral oil, which several careful tests showed to be free of acid or alkali. The outer compartment was filled with water which was maintained at a nearly constant temperature by a suitable thermal regulator. The top of the box was enclosed in a glass case, thus preventing sudden changes in the temperature of the top of the box and the bridge-wire and the accumulation of dust.

We take this opportunity to express our deep sense of obligation to the authorities of the University of Chicago, who, through the kindness of Professor W. S. Stratton, of the Ryerson Physical Lab-

oratory, placed this valuable apparatus at our disposal for this investigation.

Galvanometer.—The galvanometer was of the four-coil Thomson type, having what is practically a double metallic case so that the disturbing effects of air currents are reduced to a minimum. The dimensions of these coils were: 10 mm. external diameter, 4 mm. internal diameter, 8 mm. deep, the distance between centers of coils when mounted being 25 mm. Each coil was wound in three sections (Nos. 36, 33, and 30 B. and S. wire being used) approximately according to the theoretical curves of best winding with a total of 530 turns. The coils were covered with gold leaf and in this way connected with the surrounding metallic case to avoid electrostatic disturbances. The resistance of each coil was about 13 ohms. The sensibility of this galvanometer with all the coils in parallel (Res. = 3.2 ohms) when tested with a Wiess suspension system (two vertical magnets 27 mm. long, 1.5 mm. apart, made of small magnetized sewing needles and mounted on a thin strip of mica; total weight about 45 milligrammes) was $C = 6 \times 10^{-10}$ ampère, *i. e.*, the current required to produce 1 mm. deflection on a scale 1 meter distant when the period of a complete swing was 10 seconds. When tested with a very light suspension system weighing about 5 milligrammes (built of 10 or 12 magnets about 1 mm. long each, made of fine watch spring, tempered and strongly magnetized, total weight about 5 milligrammes) the sensibility under the same conditions was $C = 2.5 \times 10^{-10}$. All the magnets used in the suspension systems were repeatedly magnetized and boiled for several hours so that magnets of great permanency were obtained. Throughout the present investigation the Wiess suspension system was used on account of its greater freedom from external disturbances.

Barometer and Standard Meter.—The barometer was of the Regnault standard type with a tube about 1 m. in height and 25 mm. internal diameter. The lower meniscus was defined by a steel screw of known length. The barometric height was measured on the standard meter bar placed beside the mercury column by means of a cathetometer. The barometric heights as measured by the scale on the cathetometer column, merely as a check, were always found to be in fair agreement with the values found by the standard meter.

This barometer was compared with a Fortin standard barometer and the agreement (to $\frac{1}{10}$ mm.) was as close as we could read on the latter instrument.

The standard Bartel and Diedrich's meter was divided into millimeters, with the rulings on a strip of silver, imbedded in a bronze bar of square section about 21 mm. on the side. The corrections to the divisions of this meter were known from previous comparisons with a standard steel meter which had been compared with the Coast Survey and other standards by Professor Rogers.

Standardization of Resistance Box.

The resistance box, mounted as already described, was twice standardized with the greatest care. While we cannot give within the limits of the present paper the details of the methods used or the mass of figures involved, it is sufficient to state that the method was in general similar to the one described by Mr. E. H. Griffiths in Nature, November 14, 1895. The figures given below are interesting as showing the degree of accuracy attainable. It will be seen that the two calibrations gave almost identical results.

Table I.

Coils.	Nominal Values in Box Units. (Approx. 1. B. U. = .01 ohm.)	Correction (in mean B. U.) to reduce nominal values to mean box units at 20° C.		Mean.
		1st Cal.	2d Cal.	
H	5	−.0009	−.0007	−.001
G	10	+.0042	+.0036	+.004
F	20	+.0312	+.0310	+.031
E	40	+.0803	+.0799	+.080
D	80	+.0284	+.0277	+.028
C	160	−.0415	−.0430	−.042
B	320	−.0383	−.0362	−.037
A	640	−.0637	−.0635	−.063
FI	100			+.037

Calibration of Bridge-Wire.—This consists in finding the resistance of each centimeter of the bridge-wire in terms of the mean box unit at some standard temperature. In resistance box No. 7 the scale of the bridge-wire, extending in each direction from the center, is

divided into centimeters, and very approximately 1 cm. of the bridge-wire has a resistance equal to one mean box unit. The final results of the calibration are therefore given in the table below as a correction which is added to the bridge-wire reading (in cms.) to reduce this to its equivalent resistance in mean box units. The process is exactly similar to the calibration of a graduated glass tube when the length of a short column of mercury is measured in different parts of the tube. A gauge coil of manganin wire whose resistance was equal to about 2 cms. of the bridge-wire was balanced against different portions of the bridge-wire. Different parts of the bridge-wire (approx. 5 cms. long) were next balanced against H (5) coil of the resistance box, and a connection thus obtained between the bridge-wire and box unit.

TABLE II.

Correction to reduce the Bridge-Wire Reading to the Mean Box Unit at 20° C.

Bridge-Wire Reading.	Calibration Correction.	Bridge-Wire Reading.	Calibration Correction.
+0.5	−.004	−0.5	+.004
+1.0	−.007	−1.0	+.007
+1.5	−.011	−1.5	+.011
+2.0	−.015	−2.0	+.014
+2.5	−.018	−2.5	+.018
+3.0	−.022	−3.0	+.022
+3.5	−.025	−3.5	+.026
+4.0	−.028	−4.0	+.029
+4.5	−.031	−4.5	+.033
+5.0	−.034	−5.0	+.036
+5.5	−.037	−5.5	+.040
+6.0	−.041	−6.0	+.043
+6.5	−.044	−6.5	+.046
+7.0	−.047	−7.0	+.049
+7.5	−.051	−7.5	+.051
+8.0	−.054	−8.0	+.054
+8.5	−.057	−8.5	+.057
+9.0	−.060	−9.0	+.060
+9.5	−.063	−9.5	+.064

The bridge-wire vernier could be easily read to the nearest 1/100 mm., and this with considerable confidence. Observations of any constant resistance taken with the same combinations of coils and bridge-wire rarely differed by as much as 0.00002 ohm; when sev-

eral different combinations were used to measure a constant resistance, as will be seen on referring to the measurement of R_0 (the resistance of the platinum thermometer at 0° C.), the several measurements rarely differed by as much as 0.00005 ohm. This is a severe test of the accuracy of the standardization. Even these slight discrepancies are not entirely due to errors in calibration but could, we believe, be still further diminished by stirring the water in the outer tank surrounding the oil bath in which the coils are immersed. Such great accuracy in the standardization of a resistance box is, of course, easily obtained as long as we are only concerned with the relative values of the coils, which is all that we care about in the measurement of platinum temperature; but if the absolute values of the coils were required, a similar degree of accuracy would involve a vast amount of most painstaking work, mainly on account of the unscientific construction of most standard coils, which renders the accurate determination of their temperature impossible and which have undergone considerable changes in resistance with time.

For the platinum thermometer used throughout this investigation a change in temperature of 0°.001 C. corresponds to a change in resistance of about 0.00001 ohm. Owing to the great sensibility of the galvanometer changes in resistance far more minute than the vernier would indicate (*i. e.*, 0.00001 ohm) were easily detected. Advantage was taken of the high sensibility of the galvanometer by reducing the current used to measure the resistance of the coil of the platinum thermometer to 0.002 or 0.003 of an ampère, thus almost entirely eliminating any heating up of the coil by the measuring current, which was only kept on for a few seconds at most. Perhaps an equally satisfactory method of reducing to a minimum the uncertain effects of heating of the platinum coil by the measuring current would be to so regulate the current through the coil that the energy used up in heating the coil is always the same at all temperatures.

Constants of Platinum Thermometer.

Determination of R_0 (the resistance of the platinum thermometer at 0°C.).—The ice used in these determinations was that manufactured by the Diamond Ice Company of Baltimore, and was very clear and pure. The water used in this ice is first filtered; it is

then frozen in large forms (long and high, but of shallow depth) from the sides towards the center. When a thin sheet of water remains at the center, the freezing is stopped and this water drawn off, thus insuring ice of great purity, provided it is kept from contact with the freezing mixture. Conductivity tests made on water obtained from this ice show a high specific resistance.

The thermometer was inserted in a mixture of this ice (pounded very fine) and distilled water. In the first and second determinations of R_0 (Tables IV. and V.) a double-wall vessel was used, the inner vessel being nickel plated. Ice and water were placed in both compartments. The outer vessel in all the zero determinations of the platinum, as well as for the mercurial thermometers, was wrapped with a heavy boiler felt to thickness of about 4 cm.

The second and third determinations (Tables V. and VI.) were made in the same mixture of ice and distilled water, the only difference being that in the third the inner vessel was removed. The higher resistance obtained in this case (corresponding to about 0°.002 C.) strongly suggested that, in a room whose temperature is 20° C. above that of the thermometer, the effect of radiation cannot be neglected. We were at first of the opinion that the removal of the inner vessel would allow a more free circulation of the slightly warmed water coming from the top and sides, and thus the difference could be accounted for, but considering the distance of the thermometer coil from the top and sides and the compactness of the ice this does not seem possible.

Table III. is a preliminary determination of R_0 made merely to test the working of the apparatus. In this experiment ordinary ice and tap water were used. The thermal regulator was not yet adjusted and the temperature of the coils was varying quite rapidly, yet the value of R_0 is almost identical with that obtained subsequently.

TABLE III.

PLATINUM THERMOMETER IN ICE.

Time March 10 1897.	Coils.	Bridge-Wire.	Temperature of Coils.	R_0
4-10	C, D, G, H 255 Coil cor. −.011	B.-W. Reading +3.461 Cal. cor. − .025 Zero cor. − .011	20°.80 Temp. cor. +.053	258.467
4-20	C, D, F 260 Coil cor. +.017	B.-W. Reading −1.608 Cal. cor. + .012 Zero cor. − .011	20°.90 Temp. cor. +.061	258.471
4-23	C, D, F, H 265 Coil cor. +.016	B.-W. Reading −6.643 Cal. cor. + .047 Zero cor. − .011	20°.90 Temp. cor. +.062	258.471
			Mean	258.470

TABLE IV.

PLATINUM THERMOMETER IN ICE.

Time.	Coils.	Bridge-Wire.	Temperature of Coils.	R_0
12-20	C, D, F 260 Coil cor. +.017	B.-W. Reading −1.556 Cal. cor. + .011 Zero cor. − .016	20°.29 Temp. cor. +.020	258.476
12-27	C, D, G, H 255 Coil cor. −.011	B.-W. Reading +3.506 Cal. cor. − .025 Zero cor. − .016	20°.29 Temp. cor. +.019	258.473
12-32	C, D, F 260 Coil cor. +.017	B.-W. Reading −1.558 Cal. cor. + .011 Zero cor. − .016	20°.30 Temp. cor. +.020	258.474
12-40	C, D, G, H 255 Coil cor. −.011	B.-W. Reading +3.503 Cal. cor. − .025 Zero cor. − .016	20°.33 Temp. cor. +.022	258.473
12-50	C, D, F, H 265 Coil cor. +.016	B.-W. Reading −6.600 Cal. cor. + .046 Zero cor. − .016	20°.34 Temp. cor. +.023	258.470
12-55	C, D, F 260 Coil cor. +.017	B.-W. Reading −1.563 Cal. cor. + .011 Zero cor. − .016	20.°35 Temp. cor. +.024	258.473
1-05	FI, C 260 Coil cor. −.005	B.-W. Reading −1.544 Cal. cor. + .011 Zero cor. − .016	20°.37 Temp. cor. +.025	258.471
1-15	FI,D,E,F,G,H, 255 Coil cor. +.179	B.-W. Reading +3.306 Cal. cor. − .024 Zero cor. − .016	20.°38 Temp. cor. +.025	258.470
1-20	C, D, F 260 Coil cor. +.017	B.-W. Reading −1.566 Cal. cor. + .011 Zero cor. − .016	20.°39 Temp. cor. +.026	258.472
			Mean	258.472

TABLE V.

PLATINUM THERMOMETER IN ICE.

Time May 6.	Coils.	Bridge-Wire.	Temperature of Coils.	R_0
3-45	C, D, F, H 265 Coil cor. +.016	B.-W. Reading −6.589 Cal. cor. + .047 Zero cor. − .003	19°.94 Temp. cor. −.004	258.467
3-55	C, D, G, H 255 Coil cor. −.011	B.-W. Reading +3.515 Cal. cor. − .025 Zero cor. − .003	19°.90 Temp. cor. −.006	258.470
4-00	C, D, F, H 265 Coil cor. +.016	B.-W. Reading −6.588 Cal. cor. + .047 Zero cor. − .003	19°.90 Temp. cor. −.007	258.468
4-05	C, D, F 260 Coil cor. +.017	B.-W. Reading −1.550 Cal. cor. + .011 Zero cor. − .003	19°.91 Temp. cor. −.005	258.470
4-12	FI, C 260 Coil cor. −.005	B.-W. Reading −1.527 Cal. cor. + .011 Zero cor. − .003	19°.92 Temp. cor. −.005	258.471
4-20	FI, D, E, F, G, H 255 Coil cor. +.179	B.-W. Reading +3.323 Cal. cor. − .024 Zero cor. − .003	19°.93 Temp. cor. −.005	258.470
4-28	C, D, F 260 Coil cor. +.017	B.-W. Reading −1.550 Cal. cor. + .011 Zero cor. − .003	19°.94 Temp. cor. −.004	258.471
4-40	C, D, F 260 Coil cor. +.017	B.-W. Reading −1.552 Cal. cor. + .011 Zero cor. − .003	19°.90 Temp. cor. −.006	258.467
			Mean	258.469

Determination of R_1.—The resistance at 100° C., R_1, was determined in an hypsometer in which the thermometer was screened on all sides (by polished metal screens) from the effects of radiation. The hypsometer was provided with a small water manometer to indicate the excess of pressure within. To determine the atmospheric pressure the barometer already described was used in connection with the standard B. and D. scale. The temperature of the barometric column was taken by means of two Bender and Hobein thermometers of Normalglas graduated to $\frac{1}{10}$° C., whose indications were reduced to the air scale through a previous comparison with the platinum thermometer and a Tonnelot thermometer standardized at the Bureau International. The entire barometric column

TABLE VI.

PLATINUM THERMOMETER IN ICE.

Time May 6.	Coils.	Bridge-Wire.	Temperature of Coils.	R_0
5-00	C, D, F, H 265 Coil cor. +.016	B.-W. Reading −6.589 Cal. cor. + .047 Zero cor. − .003	19°.97 Temp. cor. −.002	258.469
5-10	C, D, F 260 Coil cor. +.017	B.-W. Reading −1.554 Cal. cor. + .011 Zero cor. − .003	19°.97 Temp. cor. −.002	258.469
5-15	C, D, G, H 255 Coil cor. −.011	B.-W. Reading +3.509 Cal. Cor. − .025 Zero Cor. − .003	19°.97 Temp. cor. −.002	258.468
5-20	FI, C 260 Coil cor. −.005	B.-W. Reading −1.527 Cal. cor. + .011 Zero cor. − .003	19°.98 Temp. cor. −.001	258.475
5-24	FI,D,E,F,G,H 255 Coil cor. +.179	B.-W. Reading +3.320 Cal. cor. − .024 Zero cor. − .003	19°.98 Temp. cor. −.001	258.471
5-29	FI, C 260 Coil cor. −.005	B.-W. Reading −1.527 Cal. cor. + .011 Zero cor. − .003	19°.99 Temp. cor. −.001	258.475
5-35	C, D, F 260 Coil cor. +.017	B.-W. Reading −1.550 Cal. cor. + .011 Zero cor. − .003	20°.00 Temp. cor. −.000	259.474
			Mean	258.471

was wrapped to a thickness of several centimeters with asbestos paper and cotton to ensure uniformity of temperature. The thermometers were inserted between the wrappings touching the glass of the barometer. The temperature of the standard scale was taken by another similar thermometer. A series of measurements of the barometric height were made by one observer while the other observer measured the resistance of the platinum thermometer in steam, with different combinations of coils and bridge wire. A time chart of the barometric pressure served to give the pressure at the instant when the resistance was taken. Usually the observations of the barometric height were taken as nearly simultaneous as possible with the resistance of the platinum thermometer, for the variations of the atmospheric pressure seem to take place suddenly, and not uniformly as indicated by a time chart. Moreover, the

platinum thermometer seemed to respond to the changes more rapidly than the barometer, which appeared to have an appreciable lag.

The observed barometric height, corrected for temperature and errors of graduation of the standard meter bar, was reduced to 0°C., latitude 45°, sea level. From the reduced barometric height, the temperature of the steam was obtained by interpolation in Broch's tables of the pressure of aqueous vapor recalculated from Regnault's experiments. (Trav. et Mem. du Bur. Int. des Poids et Mes., T. I.)

From the observed resistance of the platinum thermometer and the corresponding temperature of the steam, the resistance R_1 at 100°C., was deduced as follows:

For example, from the first observation in the following table, VII., we have

$$\left\{\begin{array}{l}\text{Resistance } R = 358.053 \text{ mean box units (Box No. 7).}\\ \text{Red. barometric } ht = 755.19 \text{ mm.}\end{array}\right.$$

Corresponding temperature, t, of steam, obtained from Broch's tables = 99°.822.

The correction, dt/dp, to be applied to the air temperature, t, of the boiling point of water to reduce to the boiling point at 760 mm. of mercury at 0°C., latitude 45° sea level, is, therefore, 0°.0368 per mm. of mercury.

To find the corresponding correction to be applied to the platinum temperature we have

$$t - pt = \delta\{(t/100)^2 - t/100\},$$

differentiating

$$\frac{d.pt}{dp} = \frac{d.pt}{dt}\frac{dt}{dp} = .0368\left\{1 - \delta\frac{2t - 100}{10000}\right\}.$$

We may assume an approximate value of $\delta = 1.50$ (from a previous knowledge of the constants of platinum thermometers) as sufficiently accurate for the purpose of this reduction.

If it happens that the values of δ assumed differ greatly from the final value obtained from the standardization, it may be necessary to substitute this new value of δ; we have then $\frac{dpt}{dp} = 0°.0362$ per

mm. of mercury. From the equation defining platinum temperature

$$pt = \frac{R - R_0}{R_1 - R_0} \times 100$$

we have

$$R_1 = R_0 + \frac{100}{pt}(R - R_0).$$

Reducing $t = 99°.822$ to the corresponding temperature on the platinum scale, we have $pt = 99°.826$.

$$\text{Hence } R_1 = 258.471 + 100\,\frac{(358.053 - 258.471)}{99.826}$$

$$= 358.227 \text{ mean box units.}$$

From the observations on the next page we have,

$R_0 = 258.471$ (Mean box units, at 20°C., of box No. 7),

$R_1 = 358.231$ (Mean box units, at 20°C., of box No. 7).

This gives

$$\frac{R_1}{R_0} = 1.385962.$$

Mr. E. H. Griffiths was kind enough to standardize this thermometer with great care before sending it and the values found by him (as a mean of many observations with different combinations of coils and bridge wire of box No. 6) were :

January 21, 1896, $R_1 = 358.075$ Mean box units, Box 6.

$R_0 = 258.366$ Mean box units, Box 6.

$R_s = 679.680$ (barometer at 771.35 mm.),

hence $\delta = 1.490$.

"In this determination the average temperature of the box was about 15°.4, but it was rapidly varying. Moreover the temperature correction was very large, the box being right at 20°C."

TABLE VII.

PLATINUM THERMOMETER IN STEAM.

Time May 4.	Coils.	Bridge-Wire.	Temperature of Coils.	Bar. Pres. Red. to 0° lat. 45° sea level.	Resistance at Obs. Pres.	R_1
2–37	B, E 360 Coil cor. +.043	B.-W. Reading −1.976 Cal. cor. + .014 Zero cor. − .003	19°.73 Temp. cor. −.025		358.053	358.227
2–46	B, F, G, H 355 Coil cor. −.003	B.-W. Reading +3.103 Cal. cor. − .023 Zero cor. − .003	19°.75 Temp cor. −.023		358.052	358.226
3–08	FI, C, D, F 360 Coil cor. +.054	B.-W. Reading −1.993 Cal. cor. + .014 Zero cor − .003	19°.79 Temp. cor −.020	755.19	358.052	358.226
3–20	B, E 360 Coil cor. +.043	B.-W. Reading −1.982 Cal. cor. + .014 Zero cor. − .003	19°.80 Temp. cor. −.019		358.053	358.227
3–34	FI, C, D, G, H 355 Coil cor. +.026	B.-W. Reading +3.080 Cal. cor. − .022 Zero cor. − .003	19°.80 Temp. cor. −.019	755.32	358.062	358.232
May 7.						
12–17	B, E 360 Coil cor. +.043	B.-W. Reading −1.864 Cal cor + .014 Zero cor. − .003	20.°00 Temp. cor. .000		358.190	358.228
12–25	B, F, G, H .355 Coil cor. −.003	B.-W. Reading +3.220 Cal. cor. − .023 Zero cor. − .003	20.°01 Temp. cor. +.001	758.95	358.192	358.230
12–34	B, E, H .365 Coil cor. +.042	B.-W. Reading −6.900 Cal. cor. + .049 Zero cor. − .003	20°.02 Temp. cor. +.002		358.192	358.230
May 7.						
3–55	B, E 360 Coil cor. +.043	B.-W. Reading −1.876 Cal. cor. + .014 Zero cor. − .003	20°.35 Temp. cor. +.033	759.36	358.211	358.235
4–05	B, F, G, H 355 Coil cor. −.003	B.-W. Reading +3.214 Cal. cor. − .022 Zero cor. − .003	20°.36 Temp. cor. +.033		358.219	358.236
4–15	B, E 360 Coil cor. +.043	B.-W. Reading −1.870 Cal. cor. + .014 Zero cor. − .003	20°.37 Temp. cor. +.035	759.54	358.219	358.236
4–18	"	B.-W. Reading −1.870 Cal. cor. + .014 Zero cor. − .003	20°.37 Temp. cor. +.035		358.219	358.236

$\therefore R_1 = 358.231$ mean box units Box No. 7.

January 23, 1896, $R_0 = 258.362$,

$R_1 = 358.078$,

$R_s = 679.510$ (barometer at 769.20),

hence $\frac{R_1}{R_0} = 1.38596$,

$\delta = 1.491$.

The remarkable agreement of our value of $\frac{R_1}{R_0}$ with that found by Griffiths rendered it unnecessary for us to determine the resistance of the thermometer in boiling sulphur (the third temperature usually employed) and we, therefore, accepted the value of δ, 1.491, given by Griffiths as correct.

[It is interesting to note that for the standardization given under date of January 21st, the value of

$$pt = -\frac{R_0}{R_1 - R_0} \times 100 = -259°.12,$$

which corresponds to zero resistance of the platinum thermometer, gives for the absolute zero (by using $\delta = 1.490$ in $t - pt = \delta \{(t/100)^2 - t/100)\} - 274°.44$. Similarly the standardization of January 23d gives the absolute zero $-274°.41$.]

Standardization of Platinum Thermometer.

The details of the elaborate comparisons by Mr. Griffiths, on which the constant δ of this thermometer is based, are given below (Tables VIII., IX., X.). The resistance box (No. 6) used in this standardization was similar to the one described in Nature, November 14, 1895.

"For all purposes of comparison between the results obtained here and in Baltimore, we only require the values of $\frac{R_1}{R_0}$ and δ, the magnitude of the unit used being of no consequence. I, however, give the observations in full, to show the probable order of accuracy."

January 21, 1896.

"On the above date a preliminary series of observations was made in ice, steam and sulphur vapor. The observations, however, were hurried, the resistance box was not contained in its tank, and its temperature was about 15°.50 C. Thus the temperature correction was large. I did not regard the work as anything but preliminary and will therefore only give results (the resistances are corrected for temperature of box, etc.)."

$R' = 358.480$ when barometer[1] $= 771.24$

$R_0 = 258.366$

$R_s = 679.680$ when barometer $= 771.35$

Hence we get $R_1 = 358.075$, $R_0 = 258.366$, $FI = 99.709$

$\frac{R_1}{R_0} = 1.38592$

$\delta = 1.490$

DETERMINATION OF R_0, R_1 AND FI.[2]

"We have $R' = 358.416$ when bar $= 769.39$ ∴ b. p. $= 100°.344$

$R_0 = 258.362$

$R' - R_0 = 100.054$ for dif. in temp. $= 100°.344$,

"hence mean $\frac{\delta R}{\delta t} = 0.99711$.

"Now $\frac{\delta pt}{\delta t}$ at $100° = 0.985$,

$\therefore \delta R'$ at $100° = .9971 \times .985 = 0.9821$

$\therefore \delta R'$ for $0°.344 = .9821 \times .344 = 0.338$

$\therefore R_1 = 358.416 - 0.338 = 358.078$.

"Hence $R_1 = 358.078$

$R_0 = 258.362$

$FI = 99.716$

$R_1/R_0 = 1.38596$.

[1] The barometer is in each case corrected for temperature, scale errors, and for value of g to sea-level, latitude 45°.

[2] Observations given in Tables VIII., IX., and X.

DETERMINATION OF δ.

$R_s = 679.510$ when barometer $= 769.20$ mm.

$$Pt_s = \frac{679.510 - 258.362}{99.716} = 422°.35.$$

"Now the boiling point of sulphur at 769.20 mm =

$$444°.53 + °.082 \times 9.20 = 445°.28$$

$$\therefore 445°.28 - 422°.35 = \delta\{(4.453)^2 - (4.453)\}$$

$$\therefore \delta = 1.491.$$

TABLE VIII.

PLATINUM THERMOMETER IN ICE.

Time Jan. 23.	Coils.	Bridge-Wire Reading.	Temperatures of Coils.	R_s A Box No. 6.
5-19	C, D, F, H, 265 Coil cor. −.102	B.-W. Reading −6.602 Cal. cor. + .109 Zero cor. − .078	20°.50 Cor. +.034	258.361
5-21	"	B.-W. Reading −6.600 Cal. cor. + .109 Zero cor. − .078	"	258.363
5-24	"	B.-W. Reading −6.601 Cal. cor. + .109 Zero cor. − .078	"	258.362
5-27	C, D, F 260 Coil cor. −.094	B.-W. Reading −1.526 Cal. cor. + .026 Zero cor. − 0.78	"	258.362
5-28	"	B.-W. Reading −1.524 Cal. cor. + .026 Zero cor. − .078	"	258.364
5-30	C, D, G, H 255 Coil cor. −.078	B.-W. Reading +3.546 Cal. cor. − .060 Zero cor. − .078	"	258.364
5-32	"	B.-W. Reading +3.545 Cal. cor. − .060 Zero cor. − .078	"	258.363
5-35	C, D, G 250 Coil cor. −.079	B.-W. Keading +8.626 Cal. cor − .144 Zero cor. − .078	"	258.359
5-37	"	B.-W. Reading +8.627 Cal. cor. − .144 Zero cor. − .078	"	258.360
			Mean	258.362

"*Final Results* :—

$$R_1 = 358.078$$
$$R_0 = 258.362$$
$$FI = 99.716$$

$$\frac{R_1}{R_0} = 1.38596$$
$$\delta = 1.491$$

TABLE IX.

PLATINUM THERMOMETER IN STEAM.

Time Jan. 23.	Coils.	Bridge-Wire Reading.	Temperature of Coils.	Barom. (cor'd.)	R' Box No. 6.
4-36	C, D, F, H, FI 365 Coil cor. −.179	B.-W. Reading −6.466 Cal. cor. + .106 Zero cor. − .078	20°.32 Cor. +.030	769.40	358.413
4-43	"	B.-W. Reading −6.469 Cal. cor. + .106 Zero cor. − .078	20°.38 Cor. +.035	"	358.415
4-46	C, D, F, FI 360 Coil cor. −.171	B.-W. Reading −1.394 Cal cor. + .023 Zero cor. − .078	20°.40 Cor. +.037	"	358.417
4-51	"	B.-W. Reading −1.397 Cal. cor. + .023 Zero cor. − .078	20°.42 Cor. +.039	769.45	358.416
4-55	C, D, G, FI 355 Coil cor. −.164	B.-W. Reading +3.674 Cal. cor. − .061 Zero cor. − .078	20°.45 Cor. +.042	"	358.413
4-59	"	B.-W. Reading +3.676 Cal. cor. − .061 Zero cor. − .078	20°.47 Cor. +.044	769.40	358.417
5-6	C, D, G, FI 350 Coil cor. −.156	B.-W. Reading +8.752 Cal. cor. − .146 Zero cor. − .078	20°.49 Cor. +.046	769.35	358.418
5-9	"	B.-W Reading +8.750 Cal. cor. − .146 Zero cor. − .078	20°.49 Cor. +.046	769.30	358.416
				769.39	358.416

$R' = 358.416$ mean box units (Box No 6) when the barometer is 769.39 mm.

In sulphur vapor all the precautions must be taken which are mentioned on pp. 144–147, Trans. Roy. Soc. A., 1891, vol. 182.

TABLE X.

PLATINUM THERMOMETER IN SULPHUR VAPOR.

Time Jan. 23.	Coils.	Bridge-Wire.	Temperature of Coils.	Barom (cor'd.) mm.	R' Box No. 6.
6-15	A, E 680 Coil cor. +.193	B.-W. Reading − .675 Cal. cor. + .009 Zero cor. − .078	20°.36 Cor. +.064	769.20	679.513
6-18	"	B.-W. Reading − .671 Cal. cor. + .009 Zero cor. − .078	20°.35 Cor. +.062	"	679.515
6-21	"	B.-W. Reading − .672 Cal. cor. + .009 Zero cor. − .078	"	"	679.514
6-25	A, E, H 685 Coil cor. +.185	B.-W. Reading −5.754 Cal. cor. + .095 Zero. cor. − .078	"	769.15	679.510
6-26	"	B.-W. Reading −5.755 Cal. cor. + .095 Zero cor. − .078	"	769.20	679.509
6-30	A, F, G, H 675 Coil cor. +.068	B.-W. Reading +4.527 Cal. cor. − .076 Zero cor. − .078	"	"	679.503
6-32	"	B.-W. Reading +4.528 Cal. cor. − .076 Zero cor. − .078	"	769.25	679.504
				769.20	679.510

R' = 679.510 mean box units (Box No. 6) when barometer = 769.20 mm.

RESULTS OF COMPARISONS.

In order to show in detail the method of comparison, we have selected at random the results of a single comparison of Baudin 6166 with the platinum thermometer. (Table XI.)

In columns II., III., and IV. are shown respectively the coils used, the bridge-wire reading, and temperature of coils, for the measurement of the resistance of the platinum thermometer. The corresponding stem reading on the mercurial thermometer, taken at the same instant, is shown in column VIII. The temperature of the air near the portion of the stem of the Baudin thermometer projecting from the calorimeter, and of the water in the small water jacket

around the stem, where it emerges from the calorimeter, is shown in columns IX. and X.

The resistance of the platinum thermometer is deduced from the observed quantities, shown in columns II., III. and IV., as follows: To the nominal value of the coils is applied the "coil correction" to reduce to mean box units; these are taken from Table I., and are designated "coil cor." in column II.; a further correction for the temperature of the coils must be applied to reduce to "the mean box unit of Box No. 7 at 20°C.; these corrections are applied in column IV.; to the bridge-wire reading is applied the "calibration correction" to reduce the observed reading to mean box units; to this must be further applied the "zero correction," which takes into account the amount by which the zero of the vernier differs from the zero of the scale when the bridge is balanced with the intervals C_1C_2 and P_1P_2 short-circuited and all plugs replaced with care; these corrections are applied in column III.

By the application of these corrections, taken with proper signs, we get R, the resistance of the platinum thermometer, shown in column V.

The platinum temperature, shown in column VI., is deduced from the observed value of R by substitution in the formula defining platinum temperature, remembering that $R_0 = 258.471$ and $R_1 = 358.231$ "mean box units at 20°C."

From the platinum temperature, the corresponding temperature on the air scale, shown in column VII., is deducted by the formula

$$t - pt = 1.491 \left\{ \left(\frac{t}{100}\right)^2 - \frac{t}{100} \right\}$$

In order to facilitate these reductions, a table was constructed, giving the value of $t - pt$ for every degree pt on the platinum scale.

Six determinations of the zero of Baudin 6166 gave the following results:

	mm.
	24.159
	24.147
	24.151
	24.148
	24.151
	24.154
Mean,	24.15

TABLE

COMPARISON OF BAUDIN 6166 WIT

Time May 28.	Coils.	Bridge-Wire.	Temperature of Coils.	Plat. Resist. R	Pt. Te $pt = \frac{R-}{R_1-}$
1-39	C, D, F 260 Coil cor. +.017	B.-W. Reading +3.737 Cal. cor. − .026 Zero cor. − .003	19°.64 Temp. cor. −.024	263.701	5.2
1-43	FI, C 260 Coil cor. −.005	B.-W. Reading +3.784 Cal. cor. − .027 Zero cor. − .003	19°.64 Temp. cor. −.024	263.725	5.2
1-46	C, D, F, H 265 Coil cor. +.016	B.-W. Reading −1.248 Cal. cor. + .009 Zero cor. − .003	19°.64 Temp. cor. −.025	263.749	5.2
1-54	FI, C, H 265 Coil cor. −.006	B -W. Reading −1.182 Cal. cor. + .008 Zero cor. − .003	19°.64 Temp. cor. −.025	263.792	5.3
2-00	C, D, F, G 270 Coil cor. +.021	B.-W. Reading −6.221 Cal. cor. + .044 Zero cor. − .003	19°.65 Temp. cor. −.025	263.816	5.3
2-08	C, D, F 260 Coil cor. +.017	B.-W. Reading +3.905 Cal. cor. − .027 Zero cor. − .003	19°.67 Temp. cor. −.022	263.870	5.4
2-40	C, D, F, G 270 Coil cor. +.021	B.-W. Reading −1.350 Cal. cor. + .009 Zero cor. − .003	19°.69 Temp. cor. −.022	268.655	10.2
2-44	FI, C, G 270 Coil cor. −.001	B.-W. Reading −1.316 Cal. cor. + .009 Zero cor. − .003	19°.69 Temp. cor. −.022	268.667	10.2
2-50	FI, C, H 265 Coil cor. −.006	B.-W. Reading +3.730 Cal. cor. − .026 Zero cor. − .003	19°.69 Temp. cor. −.021	268.674	10.2
2-56	C, D, F, G 270 Coil cor. +.021	B.-W. Reading −1.333 Cal cor. + .009 Zero cor. − .003	19°.71 Temp. cor. −.020	268.674	10.2
3-02	C, D, F, H 265 Coil cor. +.016	B.-W. Reading +3.711 Cal. cor. − .026 Zero cor. − .003	19°.72 Temp. cor. −.019	268.679	10.2
3-08	C, D, F, G, H 275 Coil cor. +.020	B.-W. Reading −6.360 Cal. cor. + .045 Zero cor. − .003	19°.73 Temp. cor. −.019	268.683	10.2

TABLE XI.

5166 WITH PLATINUM THERMOMETER.

Pt. Temp. $pt = \frac{R - R_0}{R_1 - R_0} \times 100$	Pt. Temp. red. to Air Scale $t - pt = \delta\left\{\left(\frac{t}{100}\right)^2 - \frac{t}{100}\right\}$	Stem Reading on 6166.	Temp. of Air near Stem.	Temp. of Water in Jacket.	Corrected Mean Stem Reading.	Corresponding Temp. on Rowland's Air Scale.
5.243	5.170	90.21				
5.267	5.193	90.49	20°.22	17°.3		
5.291	5.217	90.74				
5.334	5.260	91.43	20°.22	17°.6	91.20	
					Stem cor. −.12	
5.358	5.283	91.83			91.08	5°.522
						Cor. for rise of zero −.295
5.412	5.337	92.48	20°.22	17°.4		5°.227
	5°.242					
10.209	10.075	153.22				
10.221	10.086	153.31	20°.80	17°.92		
10.228	10.093	153.43				
10.228	10.093	153.54	20°.70	18°.02	153.46	
					Stem cor. −.09	
					−.09	
10.233	10.098	153.59			153.28	10°.361
						Cor. for rise of zero −.295
10.237	10.102	153.66	20°.75	18°.20		10°.066
	10°.091					

TABLE XI.—(*Co*

COMPARISON OF BAUDIN 6166 WITH

Time May 28.	Coils.	Bridge-Wire.	Temperature of Coils.	Plat. Resist. R	Pt. Temp $pt = \frac{R - R_0}{R_1 - R_0}$
4–25	C, D, F, G, H 275 Coil cor. +.020	B.-W. Reading +3.621 Cal. cor. − .026 Zero cor. − .003	19°.82 Temp. cor. −.013	278.599	20.176
4–40	FI, C, G, H 275 Coil cor. −.002	B.-W. Reading +3.664 Cal. cor. − .026 Zero cor. − .003	19°.82 Temp. cor. −.013	278.620	20.197
4–50	FI, C, F 280 Coil cor. +.026	B.-W. Reading −1.386 Cal. cor. + .010 Zero cor. − .003	19°.82 Temp. cor. −.013	278.634	20.211
4–55	C, D, F, G, H 275 Coil cor. +.020	B.-W. Reading +3.662 Cal. cor. − .026 Zero cor. − .003	19°.84 Temp. cor. −.012	278.642	20.219
4–58	C, D, E 280 Coil cor. +.066	B.-W. Reading −1.405 Cal. cor. + .010 Zero cor. − .003	19°.85 Temp. cor. −.011	278.657	20.235
5–00	C, D, F, G, H 275 Coil cor. +.020	B.-W. Reading +3.700 Cal. cor. − .026 Zero cor. − .003	19°.85 Temp. cor. −.011	278.680	20.258
5–38	FI, C, F, H 285 Coil cor. +.025	B.-W. Reading +3.887 Cal. cor. − .027 Zero cor. − .003	19°.91 Temp. cor. −.007	288.875	30.477
5–41	FI, C, F, G 290 Coil cor. +.030	B.-W. Reading −1.135 Cal. cor. + .008 Zero cor. − .003	19°.91 Temp. cor. −.007	288.893	30.494
5–45	C, D, E, G 290 Coil cor. +.070	B.-W. Reading −1.153 Cal. cor. + .008 Zero cor. − .003	19°.92 Temp. cor. −.006	288.916	30.518
5–50	C, D, E, H 285 Coil cor. +.065	B.-W. Reading +3.901 Cal. cor. − .028 Zero cor. − .003	19°.92 Temp. cor. −.006	288.929	30.530

II.—(*Continued.*)

5 WITH PLATINUM THERMOMETER.

Pt. Temp. $= \frac{R-R_0}{R_1-R_0} \times 100$	Pt. Temp. red. to Air Scale. $t-pt = \delta\left\{\left(\frac{t}{100}\right)^2 - \frac{t}{100}\right\}$	Stem Reading on 6166.	Temp. of Air near Stem.	Temp. of Water in Jacket.	Corrected mean Stem Reading.	Corresponding Temp. on Rowland's Air Scale.
20.176	19.938	280.18				
20.197	19.959	280.43	21.02	19.98		
20.211	19.973	280.58				
20.219	19.981	280.64	21.00	20.06	280.67	
					Stem cor. .00	
					.03	
20.235	19.997	280.91	21.08	20.18	280.64	20°.232
						Cor. for rise of zero −.295
20.258	20.020	281.25	21.02	20.21		19°.937
	19°.978					
30.477	30.163	410.59				
30.494	30.180	410.85	21°.20	21°.60	411.00	
					Stem cor. +.09	
					+.43	
30.518	30.204	411.16	21°.32	21°.80	411.52	30°.458
						Cor for rise of zero −.295
30.530	30.216	411.39	21°.05	22°.00		30°.163
	30°.191					

Rowland's zero (Table XX., p. 116, Proc. Am. Acad. of Arts and Sciences, 15, 1879) was 20.43.

Temperatures by Baudin 6166 are reduced as follows: To the mean stem reading is applied the stem correction which is divided into two parts, one portion of the stem extending from 28 mm. to 99 mm. being assumed to be at the temperature of the water in the surrounding jacket, the remainder of the stem at the temperature of the surrounding air. This correction is applied in column XI.; from the corrected stem reading, the corresponding temperature on Rowland's air thermometer is obtained from the results of his comparisons of these thermometers with the air thermometer given in his Tables XVIII., XIX. and XX.; the temperatures thus obtained must be further corrected by the rise of the zero since Rowland's comparisons; this is shown in Column XII., which gives the mean temperature on the Rowland air scale (obtained through 6166) corresponding to the mean temperature on the Callendar-Griffiths' air scale given in column VII.

These results are also plotted in the form of a curve in Fig. 4.

The results of these comparisons can best be shown by means of the accompanying curves.

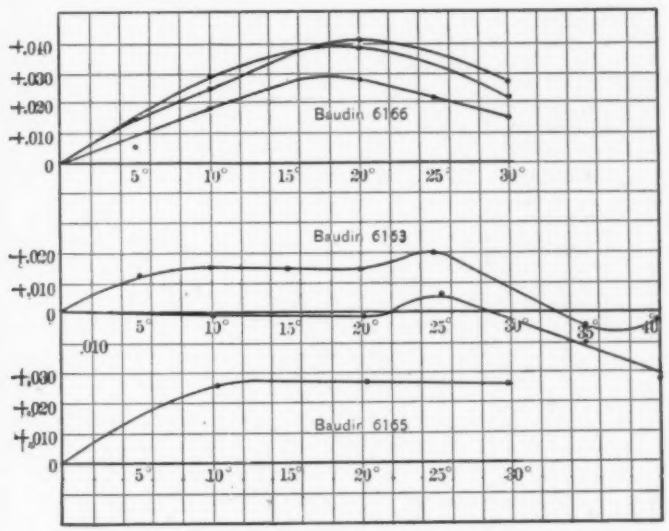

Fig. 3.

Fig. 3 shows the result of each of the independent comparisons

of Professor Rowland's Baudin thermometers with the platinum resistance thermometer. Abscissæ represent temperature on the centigrade scale, and ordinates the corresponding corrections that must be added to Rowland's air scale, as determined by the Baudin thermometers, to reduce to the Callendar-Griffiths' air scale. The almost constant difference between the curves of May 18th and May 24th suggests at once a constant error affecting the entire series of one or the other comparisons. This difference is, however, not of any great importance, for in the determination of the mechanical equivalent we are practically only concerned with temperature ranges over the interval 5° to 35°, and these are practically identical on either curve. After looking over the results we have attributed these differences to a slight error in the value of the temperature coefficient of the coils of the box, which, on account of the high temperature of the room, could not be kept at 20° in the comparisons of May 24th and May 21st; this conclusion is further strengthened by the fact that the comparison of May 28th when the box was again near 20° practically coincides with that of April 10th.

A résumé of all the comparisons between the platinum and Baudin thermometers is also given in the following table.

TABLE XIII.

Date, 1897.	No. of Observations.	Temp. on the Callendar-Griffiths Air Scale.	Temp. on Rowland's Air Scale.	Difference Δ
Comparison of Platinum Thermometer with Baudin 6163.				
May 18	5	5°.261	5°.249	+0°.012
	6	9 .937	9 .922	+0 .015
	6	15 .340	15 .325	+0 .015
	6	20 .066	20 .052	+0 .014
	4	24 .881	24 .861	+0 .020
	5	34 .916	34 .921	−0 .005
	4	40 .051	40 .052	−0 .001
Comparison of Platinum Thermometer with Baudin 6163.				
May 24	6	9°.997	9°.998	−0°.001
	6	20 .055	20 .056	−0 .001
	6	25 .228	25 .222	+0 .006
	6	34 .899	34 .909	−0 .010
	6	40 .283	40 .305	−0 .022

Comparison of Platinum Thermometer with Baudin 6166.

April 10	5	10°.873	10°.844	+0°.029
	5	19 .650	19 .611	+0 .039
	5	30 .136	30 .114	+0 .022

Comparison of Platinum Thermometer with Baudin 6166.

May 21	5	5°.131	5°.127	+0°.004
	6	9 .869	9 .852	+0 .017
	6	14 .899	14 .871	+0 .028
	7	19 .951	19 .923	+0 .028
	6	24 .987	24 .965	+0 .022
	6	30 .226	30 .212	+0 .015

Comparison of Platinum Thermometer with Baudin 6166.

May 28	6	5°.242	5°.227	+0°.015
	6	10 .091	10 .066	+0 .025
	6	19 .978	19 .937	+0 .041
	4	30 .191	30 .163	+0 .028

Comparison of Platinum Thermometer with Baudin 6165.

April 14	5	10°.129	10°.104	+0°.025
	5	20 .274	20 .247	+0 .027
	5	29 .735	29 .709	+0 .026

Comparison of Platinum Thermometer with Tonnelot 11801.

This comparison was made by Dr. W. S. Day and the authors, in order to bring the results of the different standards of thermometry into comparison, and thus, if possible, to account for the differences in the values of the capacity for heat of water obtained by different observers.

The comparison was carried out in a tank[1] especially designed by Dr. Day for the comparison of mercurial thermometers in a horizontal position. This tank consists of a long rectangular copper box 98 cm. long, 21 cm. wide, and 23 cm. deep, with a movable cover, holding a piece of plate glass, through which the stem readings of the thermometers could be observed by means of a micrometer telescope which slides along parallel ways supported from the cover of the tank. Within this copper tank was another small brass box, 10.5 cm. long, 8 cm. wide, and 5 cm. deep, in which were placed,

[1] See W. S. Day, Phys. Rev., Apr. 1898.

side by side, the bulbs of the thermometers to be compared. This box shielded the bulbs from the effects of convection currents, and in this way the thermometers were kept at a constant temperature for a considerable time, for it required an interval of some minutes for a very appreciable change in the temperature of the large mass of surrounding water to produce a minute change in the temperature of the enclosed water, owing to the absence of convection currents. This brass box was provided with two movable lids on the top, and two on the bottom, which could be opened and closed from the outside of the tank. The large tank was provided with three paddle wheels, operated from the outside, which produced efficient stirring. The entire tank was then placed inside a wooden box, the interspace between the walls of the tank and box being loosely filled with cotton wool. The greater portion of the lid, with the exception of the plate glass, was covered with a layer of thick felt. Tne tank and its contents could be raised to any desired temperature by allowing steam to flow through a copper tube coiled along the bottom of the tank.

As the platinum thermometer is peculiarly adapted to show minute changes in temperature, the efficiency of this tank was exposed to a severe test in these comparisons. It was thus ascertained that, if the lids of the inner brass box were opened, the entire contents of the tank thoroughly stirred, and the lids then tightly closed, the temperature would remain constant to within 0°.001 for at least several minutes, for a difference of 20° C. between the temperature of the room and the water in the tank.

CONSTANTS OF TONNELOT 11801.

Tonnelot's 11800, 11801 and 11811 were made of French "verre dur," with transparent stems divided into tenths of a degree, about December, 1895. They were subsequently sent to the Bureau International, where a most exhaustive study of them was made by M. Guillaume, and comparisons made at several different temperatures with their standards. These standards have been compared by M. Chappuis with the standard nitrogen and hydrogen thermometer (Trav. et Mem. du Bur. Int. des Poids et Mesures, T. VI., 1888), so that the scale of these thermometers can at once be reduced to the nitrogen or hydrogen scale.

We can only briefly outline the methods used at the Bureau International for the standardization of thermometers, for to do full justice to their beautiful and painstaking researches on thermometry would not lie with the limits of the present communication.

The thermometers are first roughly examined for uniformity of bore and graduation. If these are not sufficiently accurate the thermometer is rejected. If these are found satisfactory, a *calibration* is next made by measuring the lengths of suitable columns of mercury in different portions of the scale. This determines the correction that must be applied to certain "principal points" of the scale, to reduce the scale reading to what the reading would have been if the bore had been perfectly uniform. In these thermometers, whose range includes 0° and 100°, the corrections are usually so calculated that the calibration corrections for 0° and 100° are zero.

The next constants determined are the external and internal pressure coefficients, usually denoted by β_e and β_i. The *external pressure coefficient* is determincd by subjecting the thermometer to known variations in pressure and observing the corresponding change in the stem reading. The external pressure coefficient, β_e, is then the change in degrees produced by a change in the external pressure of one mm. of mercury. The *internal pressure coefficient*, β_i is deduced from β_e by adding 0°.0000154, a quantity depending on the difference of compressibility of mercury and "verre dur."

The *fundamental interval, i. e.*, the number of scale divisions between 0° and 100°, is then determined by observing the "fixed points" (in steam and in ice) of the thermometer. This gives the valuc of the degree or the scale division (if the graduation is arbitrary) on the centigrade scale.

From these known constants the temperature on the centigrade scale of this particular thermometer can be determined. But a scale of temperature defined in this way would vary with each different specimen of glass used, and would not be exactly the same even for the same kind of glass. The mean scale of a number of French hard glass thermometers kept at the Bureau International, and which have been compared directly with the gas thermometers (H, N, and CO_2) defines temperature on the French hard glass ("verre dur") scale. The slight deviations of the hard glass scale of any other

Tonnelot thermometer which is compared with these standards is observed, and these corrections serve to reduce the indications of this thermometer to the mean French hard glass scale.

Method of Comparison.

The Tonnelot thermometer was adjusted with its stem in a horizontal position in the comparison tank, and with its bulb near the center of the inner brass box. The micrometer telescope was then adjusted by means of a level so that its axis of collimation was perpendicular to the stem of the thermometer. When these adjustments had been made no certain differences could be detected between observations made with the divisions in front of the stem and those made with divisions back of the stem. The glass stem of the platinum thermometer passed from the outside through the sides of the wooden box and comparison tank into the brass box, where its coil was almost in contact with the bulb of the mercury thermometer. Before taking an observation, the top and bottom of this box was opened and the entire contents of the tank thoroughly stirred; the lids were then tightly closed and a series of observations taken. A single series included a setting on the division below the meniscus, on the meniscus, and on the division above the meniscus, repeated three times, but in inverse order. No certain variation of temperature was ever detected during the time required to take a series of observations. The box containing the bulbs was then opened, the water again thoroughly stirred, the box closed and a similar series of observations taken. If the temperature of the water in the tank was below that of the room, after each stirring there was a slight rise in temperature; if the temperature of the water in the tank was above that of the room, there would be a slight decrease after each stirring; steam was therefore passed through the heating coil to make sure of a rising meniscus. Usually four such series of observations were taken at each temperature. Immediately after the comparisons at one temperature were completed, the Tonnelot was removed from the tank and its zero quickly determined (in a mixture of pure ice and distilled water) before any recovery of the zero shall have taken place. In the zero determinations the thermometer was adjusted vertical and the axis of collimation of the microm-

eter telescope horizontal. Occasional barometer readings were taken to reduce the indications of the Tonnelot thermometer to standard pressure.

While one observer was taking observations on the Tonnelot thermometer, another observer was taking simultaneous observations of the resistance of the platinum thermometer.

Results of Comparison.

In order to show in detail the methods used, etc., we give below the reduction of a single series of observations at one temperature.

Observations:

Platinum Thermometer	Coils FI, C, F.			
	Bridge-wire reading = — 1.015			
	Temperature of coils 20°.31			
		Micrometer readings.		
		(1)	(2)	(3)
Tonnelot Thermometer No. 11801.	20°.4	15.587	15.579	15.586
	Meniscus	15.109	15.102	15.100
	20°.5	14.995	14.995	14.989
	Barometer (brass scale)			762.0 mm.
	Temperature,			19°.6
	Zero Determination.			
	0°.0	12.610	12.612	12.605
	Meniscus	12.428	12.430	12.426

Reductions:

		Zero.
Stem reading of Tonnelot 11801	20.482	+ 0.031
Calibration correction	+ 0.004	0.000
Correction for external pressure due to 57 mm. of water + excess of atmospheric pressure	— 0.001	— 0.001
Internal pressure correction thermometer horizontal in comparison tank, vertical in zero tank	0.000	+ 0.008
		+ 0.038
Zero correction	— 0.038	
	20.447	
Correction to fundamental interval . . .	0.000	
Temperature on scale of 11801	20°.447	
Correction M, to reduce to mean French hard glass scale	— 0.006	
Temperature on mean hard glass scale . .	20°.441	
Correction to hydrogen scale	— 0.086	
Correction to nitrogen scale	— 0.076	
Temperature on hydrogen scale	20°.355	

Temperature on nitrogen scale	20°.365
Coils FI, C, F = ...	280.
Coil corrections (Table I.)	+ 0.026
Temperature correction	+ 0.023
Bridge-wire reading	— 1.015
Calibration correction (Table II.)	+ 0.007
Zero correction of bridge-wire	— 0.006
	279.035 mean box units.

$$\therefore pt = \frac{279.035 - 258.471}{358.231 - 258.471} \times 100$$

$$= 20°.613$$

$$\therefore t = 20°.372 \text{ (air scale).}$$

The results of the two independent series of comparisons of the Platinum Thermometer with Tonnelot 11801, are given in the following table (XII.).

TABLE XII.

COMPARISON OF PLATINUM THERMOMETER AND TONNELOT 11801.

Tonnelot 11801 Nitrogen Scale.	Plat. Therm. Air Scale.	Δ (Cor. to reduce Pt.-Air Scale to Paris Nitrogen Scale.)	Tonnelot 11801 Nitrogen Scale.	Plat. Therm. Air Scale.	Δ (Cor. to reduce Pt.-Air Scale to Paris Nitrogen Scale.)
7°.804	7°.800	+.004	10°.197	10.186	+.011
12.941	12.941	.000	20.519	20.523	−.004
18.332	18.327	+.005	23.400	23.391	+.009
23.106	23.114	−.008	25.568	25.571	−.003
33.703	33.704	−.001	30.004	30.005	−.001
39.999	40.001	−.002	43.817	43.820	−.003

Fig. 4 gives the final correction curves for each of the Baudin thermometers. The ordinates of curves I. (mean of the individual comparisons shown in Fig. 3) give the corrections that must be added to Rowland's air scale, as determined from the Baudin thermometers, to reduce to the Callendar-Griffiths' air scale. To pass from the air scale to the absolute scale of temperature use was made of Rowland's Table XVII. (p. 114, Proc. Am. Acad., 15, 1879). In this way curves II. were obtained which give the corrections to reduce Rowland's absolute scale to the Callandar-Griffiths' air scale.

The results of the comparison of the platinum resistance thermometer with Tonnelot 11801, which had been standardized at the Bureau International, are shown in curve A, in which ordinates rep-

resent the corrections that must be added to the Callendar-Griffiths' air scale to reduce to the Paris nitrogen scale as given by Tonnelot 11801. The close agreement of these two scales is a strong confirmation of the accuracy of the platinum-air interpolation formula. Indeed such close agreement must be partially fortuitous, as we have certainly no right to expect so close an agreement, considering the difficulties of gas thermometry.

Curves III., whose ordinates must be added to Rowland's absolute scale to reduce this to the Paris nitrogen scale, were then obtained by combining curves II. and curve A.

Curves IV. give the corrections that must be added to Rowland's absolute scale to reduce this to the Paris hydrogen scale; these curves are obtained from curves III. by making use of the relation of the hydrogen and nitrogen scales of temperature as determined by the experiments of M. Chappuis (Guillaume, Thermometrie de Précision, p. 258).

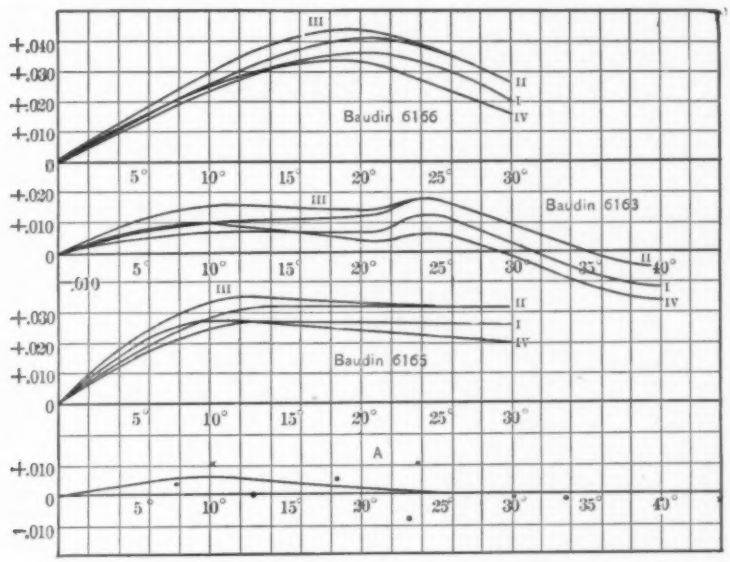

Fig. 4.

The corrections to reduce the readings of the Baudin thermometers (when referred to Rowland's absolute scale) to the Paris nitrogen and hydrogen scales of temperature, as obtained from curves III. and IV., Fig., 4 are given in the following table (XIV.).

TABLE XIV.

CORRECTIONS TO ROWLAND'S BAUDIN THERMOMETERS.

Temp. ° C.	Baudin 6163.		Baudin 6165		Baudin 6166.	
	Cor. to Paris Nitrogen Scale.	Cor. to Paris Hydrogen Scale.	Cor. to Paris Nitrogen Scale.	Cor. to Paris Hydrogen Scale.	Cor. to Paris Nitrogen Scale.	Cor. to Paris Hydrogen Scale.
1°	+.002	+.002	+.006	+.005	+.004	+.003
2°	.004	.003	.010	.009	.007	.005
3°	.006	.005	.015	.013	.010	.008
4°	.009	.007	.019	.016	.013	.010
5°	.011	.008	.022	.019	.016	.013
6°	.013	.009	.025	.022	.019	.015
7°	.014	.010	.027	.024	.022	.017
8°	.015	.011	.030	.026	.025	.019
9°	.015	.011	.032	.028	.027	.021
10°	.016	.010	.034	.028	.030	.024
11°	.015	.009	.035	.028	.032	.025
12°	.015	.009	.036	.028	.034	.027
13°	.015	.007	.036	.028	.037	.029
14°	.015	.007	.036	.027	.038	.031
15°	.015	.007	.036	.027	.040	.032
16°	.015	.006	.035	.026	.041	.033
17°	.015	.006	.035	.026	.042	.034
18°	.014	.005	.035	.025	.043	.034
19°	.014	.005	.035	.025	.043	.034
20°	.014	.004	.034	.024	.043	.033
21°	.015	.004	.033	.023	.042	.031
22°	.016	.004	.033	.023	.041	.029
23°	.017	.005	.033	.022	.039	.028
24°	.017	.006	.033	.021	.037	.026
25°	.017	.006	.032	.021	.035	.024
26°	.016	.005	.032	.021	.034	.022
27°	.015	.003	.032	.021	.032	.021
28°	.013	.002	.032	.021	.030	.019
29°	.011	.000	.032	.021	.028	.017
30°	.009	−.002	.032	.021	.027	.016
31°	.007	−.004				
32°	.005	−.005				
33°	.003	−.007				
34°	.002	−.009				
35°	.000	−.011				
36°	−.002	−.013				
37°	−.003	−.015				
38°	−.004	−.016				
39°	−.005	−.016				
40°	−.005	−.016				
41°	−.006	−.017				

Recalculations of Mechnical Equivalent.

Reduction to the Paris Nitrogen Scale—Rowland's values of the mechanical equivalent, as expressed in terms of the rise of temperature of water, are deduced from an equation of the following form:

$$J_{T^\circ} = \frac{W}{(T^\circ + 5^\circ) - (T^\circ - 5)},$$

where W is equal to the energy in ergs required to raise the temperature of the water from $T^\circ - 5^\circ$ to $T^\circ + 5^\circ$.

Hence, if C_{20} and C_{10} are the corrections that have to be applied to the temperatures indicated by the Baudin thermometers to reduce these to the Paris Nitrogen Scale, we shall have for the value of J_{15° reduced to the nitrogen scale,

$$J^N_{15^\circ} = \frac{W}{(20^\circ + C_{20}) - (10^\circ + C_{10})} = \frac{W}{10} \frac{1}{1 + \frac{C_{20} - C_{10}}{10}}$$

$$= J_{15^\circ} \frac{1}{1 + \frac{C_{20} - C_{10}}{10}} = J_{15} \left(1 + \frac{C_{10} - C_{20}}{10} \right) \text{ approximately}$$

$$= J_{15^\circ} K_{15^\circ} \text{ where } K_{15^\circ} = \left(1 + \frac{C_{10} - C_{20}}{10} \right).$$

The values of K (the factor to reduce Rowland's values of the equivalent to the Paris Nitrogen Scale) were computed, from the corrections found by the preceding comparisons, for each degree centigrade for each of the Baudin thermometers.

The results of Rowland's experiments on the mechanical equivalent of heat are summarized on pp. 192–196 Proc. Am. Acad., 15, 1879. At any one temperature the values of J were averaged for each thermometer separately, and each mean was then reduced to the nitrogen scale by the proper correction factor K. The general mean value of J at that temperature was then obtained by taking the mean of the values given by each thermometer, giving to each thermometer a weight equal to the number of experiments performed with that thermometer at the given temperature. These

values of J, corrected as indicated above, were then plotted and the values obtained from the resulting smooth curve were taken as the final values of J on the nitrogen scale. For the sake of comparison the original as well as the corrected values of J are given in the following table.

TABLE XV.

Temperature.	Rowland's values of J (absolute scale) on the C.G.S. System.	Rowland's values of J reduced to the Paris nitrogen scale on the C.G.S. System.	Temperature.	Rowland's values of J (absolute scale) on the C.G.S. System.	Rowland's values of J reduced to the Paris nitrogen scale on the C.G.S. system.
7°	4.207×10^7	4.200×10^7	22°	4.176×10^7	4.178×10^7
8°	4.204	4.198	23°	4.175	4.177
9°	4.202	4.196	24°	4.174	4.177
10°	4.200	4.195	25°	4.173	4.176
11°	4.198	4.193	26°	4.172	4.175
12°	4.196	4.192	27°	4.171	4.175
13°	4.194	4.190	28°	4.171	4.174
14°	4.192	4.189	29°	4.170	4.174
15°	4.189	4.187	30°	4.171	4.175
16°	4.187	4.186	31°	4.171	4.175
17°	4.185	4.184	32°	4.171	4.176
18°	4.183	4.183	33°	4.172	4.175
19°	4.181	4.182	34°	4.172	4.176
20°	4.179	4.181	35°	4.173	4.177
21°	4.177	4.179	37°	4.173	4.178

These results are also plotted in Fig. 5. For the sake of comparison the electrical determinations of the mechanical equivalent by Griffiths and by Schuster and Gannon are also shown on the same figure.

CONCLUSIONS.

These comparisons would seem to show that Baudin 6163, when its indications are reduced to the absolute scale by means of Rowland's tables, reads lower than the Paris nitrogen scale over the range 0° to 35°, these differences amounting to 0°.016 at 10°, 0°.014 at 20°, and 0°.009 at 30°. The corrections to reduce to the Paris hydrogen scale and the Callendar-Griffiths' air scale are respectively, 0°.010 and 0°.010 at 10°, 0°.004 and 0°.012 at 20°, and −0°.002 and 0°.009 at 30°. Baudin 6166, when its indications are reduced to the absolute scale, reads lower than the Paris nitro-

gen scale throughout the range 0° to 31°; the corrections are 0°.030 at 10°, 0°.043 at 20°, and 0°.027 at 30°. The corrections to reduce to the Paris hydrogen scale and the Callender-Griffiths' air scale are respectively 0°.024 and 0°.025 at 10°, 0°.033 and 0°.041 at 20°, and 0°.016 and 0°.027 at 30°. The indications of Baudin 6165, when reduced to the absolute scale, are about 0°.035 too low throughout the range 10° to 30°.

As will be seen from the curves for the "capacity of heat for water" shown in Fig. 5, the changes in Rowland's values are small,

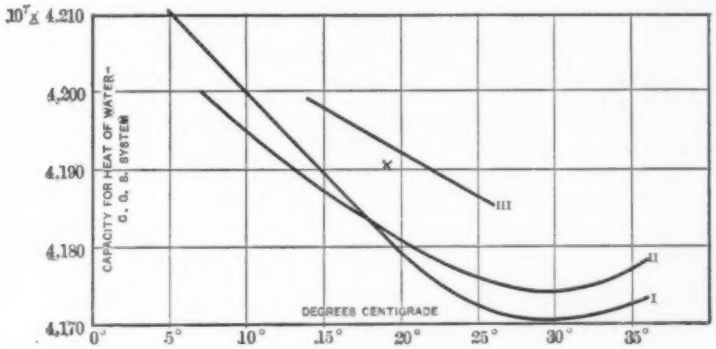

Fig. 5.

amounting to a decrease of about one part in 850 at 10° C., the value at about 18° remaining unchanged, while those at 20° and 25° are respectively increased by about 1 in 2100 and 1 in 1400. The variation between 15° and 25°, of the specific heat of water with temperature when Rowland's values are referred to the Paris nitrogen scale, is practically identical with that given by Griffiths' curve. This suggests at once that an explanation of the differences between the mechanical and electrical determinations of the mechanical equivalent must be sought in the energy measurements.

In Rowland's experiments an error in the energy measurements may be due either to an error in the diameter of the torsion wheel of his calorimeter or to the system of weights employed. Inasmuch as the diameter of the torsion wheel was measured many times by comparison with two standard meter bars, each of which had been compared with the Coast Survey and other standards, the possibility

of an error sufficient to account for the observed differences between the mechanical and electrical determinations must be sought elsewhere. Turning to the question of the weights used in these experiments we see that it is not necessary that they be correct absolutely with the standards, if they are only relatively correct, as the formula for the mechanical equivalent contains a weight in both numerator and denominator; on the other hand, if they were not correct relatively, we should hardly expect to find the almost constant difference between the determinations of Griffiths and those of Rowland (reduced to nitrogen scale) throughout the range 15° to 25°. The evidence accumulated thus far would, we believe, suggest, as a possible explanation of these differences, a still undiscovered error in the system of electric units employed. This, indeed, in the light of the enormous and painstaking work which serves as the basis of these units, is not probable, but, considering the difficulties encountered in the standards of electro-motive force and current, it is not altogether impossible. The enormous experimental evidence which served as the basis of the international ohm, together with recent confirmation of the accuracy of this unit, renders it almost certain that the difference need not be looked for in this direction.

At the Toronto meeting of the British Association, before which an abstract of this paper was read, the Committee on Electric Standards received an appropriation for the redetermination of the electro-chemical equivalent of silver and the absolute electro-motive force of the Clark cell, and their results are awaited with much interest. In the meanwhile the results of a determination of these quantities, made by Dr. Kahle at the Reichsanstalt, have been published (Zeitschr. f. Instk., 18, pp. 229, 267, 1898; Wied. Ann., 59, p. 532, 1896; Wied. Ann., 67, p. 1, 1899). His final value of the e. m. f. of the German H. standard form of Clark cell was

$$E_{15^\circ} = 1.4325 \text{ volts.}$$

By making use of a previous comparison between the Cambridge standard and the German H. standard (B. A. Report, 1892), the Cambridge standard, when reduced in accordance with Kahle's value, becomes

$$\text{Cambridge}_{15^\circ} = 1.4329 \text{ volts.}$$

The value of the Cambridge standard (upon which is based Griffiths' values of the capacity for heat of water) as determined by Glazebrook and Skinner (Phil. Trans. A., 1892) was

$$\text{Cambridge}_{15^\circ} = 1.4342 \text{ volts.}$$

As has already been shown by Dr. F. A. Wolff (Johns Hopkins Univ. Circular, June, 1898) when the values of the e. m. f. of the Clark cell as found by Kahle are applied to Griffiths' values of the "capacity for heat of water," as well as those of Schuster and Gannon, they are brought into very fair agreement with those of Rowland as corrected by the results of our comparisons (differing by about 1 part in 1400). This is shown by the following table:

		Old.	Corrected.
At 15°	Rowland (air scale)	4.189×10^7	4.187×10^7
	Griffiths (N. Scale)	4.198×10^7	4.190×10^7
At 20°	Rowland	4.179×10^7	4.181×10^7
	Griffiths	4.192×10^7	4.184×10^7
	Schuster & Gannon (N. Scale) at 19°.1	4.1905×10^7	4.185×10^7
At 25°	Rowland	4.173×10^7	4.176×10^7
	Griffiths	4.187×10^7	4.179×10^7

Dr. Guthe has kindly informed me of the result of a redetermination of the electro-chemical equivalent of silver, which was carried out during the past year by Professor Patterson[1] and himself at the University of Michigan. Their final value of the electro-chemical equivalent of silver is 0.0011193 grammes per ampère per second for a "used" solution. This is very near the value found by Kahle for a used solution; for a "fresh" solution Kahle found 0.0011182.

A recalculation of the e. m. f. of the Cambridge standard Clark cell as determined by Glazebrook and Skinner, if we use the value of the equivalent found by Guthe and Patterson, is

$$\text{Cambridge}_{15^\circ} = 1.4327 \text{ volts}$$

which is in very close agreement with the value given by Kahle.

[1] The results of this investigation have since been published in the PHYSICAL REVIEW, December, 1898, p. 257.

If Griffiths' values are corrected in accordance with this determination the agreement with Rowland's corrected values is very close, they being greater by about 1 part in 2300 throughout the range 15° to 25°.

In conclusion, the authors wish to express their sincere appreciation of Professor Rowland's kindness in allowing the use of his thermometers for these comparisons, and their many obligations to both Professor Rowland and Professor Ames for their frequent advice and assistance throughout the course of this work; and also to Dr. W. S. Day for so freely placing his apparatus at our disposal, and his many acts of kindness.

JOHNS HOPKINS UNIVERSITY, BALTIMORE.

THE ELECTROMOTIVE FORCE OF POLARIZATION OF ELECTROLYTIC CELLS FOR VERY SMALL CURRENT DENSITIES.

By Barry Mac Nutt.

THE power spent in an electrolytic cell may be divided into three parts: (*a*) that part which is proportional to the square of the current passing through the cell, and which appears as heat throughout the electrolyte. This may be written Ri^2, where the proportionality factor R is called the resistance of the cell. (*b*) that part which appears at the electrodes in the form of heat due to an irreversible thermo-dynamic process which accompanies the deposition of the ions. This may be written $\varphi(i)$ inasmuch as it is a function of the current. (*c*) that part ei, proportional to the current, which is spent in accomplishing reversible chemical effects. The proportionality factor e is the counter e. m. f. of the cell.

Suppose a current is pushed through an electrolytic cell by an e. m. f. E; then from the above the equation of power will be:

$$Ei = Ri^2 + \varphi(i) + ei,$$

or

$$E = Ri + \frac{\varphi(i)}{i} + e.$$

The action in the cell upon which the expenditure of power Ri^2 depends is an irreversible thermo-dynamic action, and the e. m. f. Ri changes sign with the current and approaches zero with the current. The action in the cell upon which the expenditure of power $\varphi(i)$ depends is an irreversible thermo-dynamic action; the e. m. f. $\frac{\varphi(i)}{i}$ changes sign with the current but is not known to approach zero with the current. The fictitious resistance $\frac{\varphi(i)}{i^2}$ is sometimes called the transition resistance ("Uebergangswiederstand").

It is the purpose of this paper to investigate the e. m. f. $\frac{\varphi(i)}{i}$ for small values of the current, or, indeed, of the current densities. This work has an important theoretical bearing as follows: If the action which takes place in an electrolytic cell for infinitesimal currents is not thermodynamically reversible $\left(\frac{\varphi(i)}{i} = 0 \text{ when } i = 0\right)$ then the ordinary principles of thermo-dynamics are not applicable to the electrolytic cell.

In an electrolytic cell having two zinc electrodes immersed in zinc sulphate, two copper electrodes immersed in copper sulphate, or two silver electrodes immersed in silver nitrate, if the electrodes are of precisely the same chemical composition and if there is no concentration of the electrolyte at the two electrodes, then the counter e. m. f. e of the cell is zero. Now as that part of the e. m. f. of an electrolytic, Ri, is proportional to the current, it may be neglected for very small currents; and if e is zero as mentioned above, the e. m. f. E would be simply $\frac{\varphi(i)}{i}$ and this e. m. f. may thus be measured.

As $\frac{\varphi(i)}{i}$ is a function of time as well as of current, experiments were made so as to determine its dependence on both.

EXPERIMENTAL WORK.

Three cells were constructed: one with zinc electrodes in a 10% solution of zinc sulphate; another with copper electrodes in a 10% solution of copper sulphate, and a third with silver electrodes in a 10% solution of silver nitrate. These solutions were covered with a layer about one-half an inch deep of heavy paraffin oil. The cells themselves were put in a double box with a thick layer of sawdust between the boxes in order to keep the temperature as nearly constant as possible. In performing the experiments the following observations were taken:

After making a great number of preparatory observations in order to eliminate to as great an extent as possible all instrumental and other errors, the cells were allowed to stand with no current

passing through them for twenty-four hours, then the maximum current was run through them and their e. m. f.'s observed at intervals during the succeeding twenty-four hours, keeping the current constant. When the e. m. f. became constant the current was reduced, and again time-e. m. f. readings were observed, as before. This was repeated until zero current was reached, and these results gave the data from which the time-e. m. f. curves were plotted (Figs. 1, 2 and 3). The current was then increased by the same

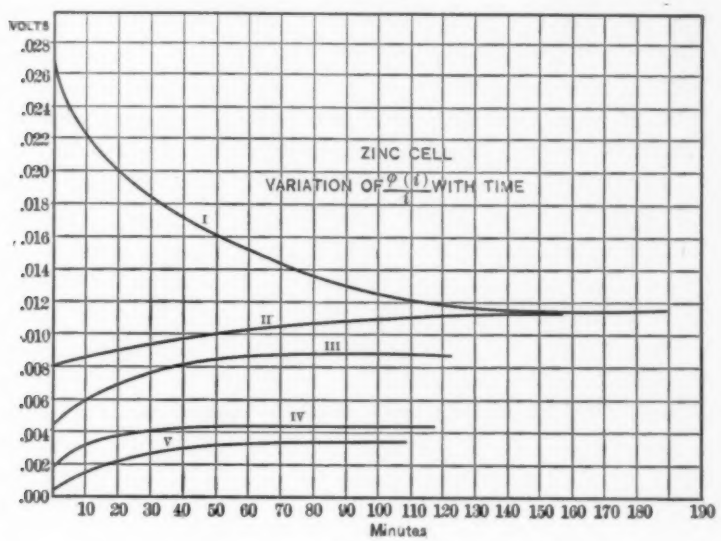

Fig. 1.

steps to its maximum value, taking observations now only of current and e. m. f., leaving a proper interval of time between observations in order to allow the e. m. f. to become steady. These intervals were determined from the preceding set of observations. The current was again reduced by the same steps to zero, and the data thus obtained gave the current e. m. f. curves (Figs. 4, 5 and 6).

Curves marked I in Figs. 1, 2 and 3, show the variation of $\frac{\varphi(i)}{i}$ with time when the maximum current used was passed through the respective cells; the cells having stood in each case with no current passing through them for twenty-four hours.

Curves marked II, Figs. 1, 2 and 3, show the variation of $\frac{\varphi(i)}{i}$ with time when the current was changed from the maximum value used when curves I were plotted to the value of .001 ampères per

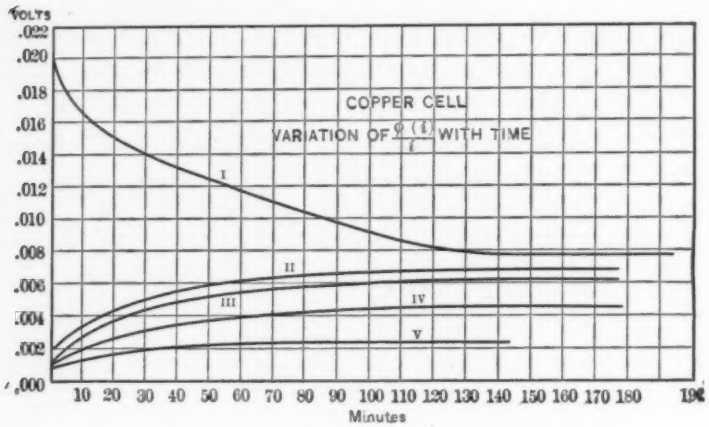

Fig. 2.

square meter. (It was necessary to interrupt the current for a few seconds when this change was made.) Curves III, IV and V,

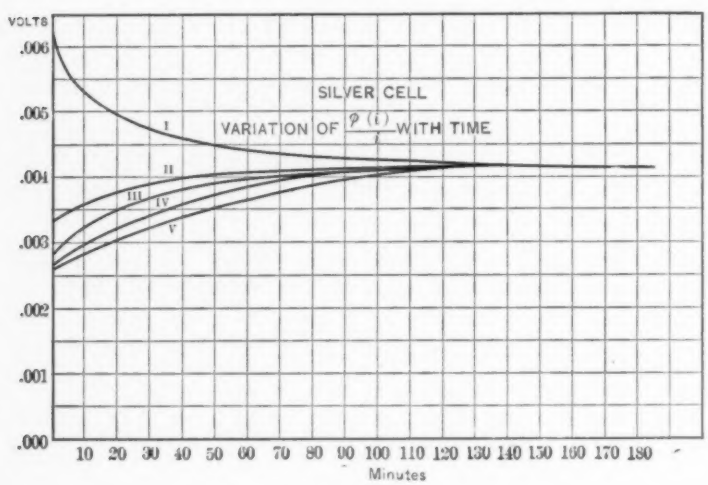

Fig. 3.

Figs. 1, 2 and 3, give the variation of $\frac{\varphi(i)}{i}$ with time for three smaller values of the current corresponding to three smaller values of the initial e. m. f.

These time-e. m. f. curves show, that when the cells have stood with no current passing through them for twenty-four hours and a constant current is then passed through them, the polarization e. m. f. $\frac{\varphi(i)}{i}$ varies *inversely* with the time, but if a current has been flowing

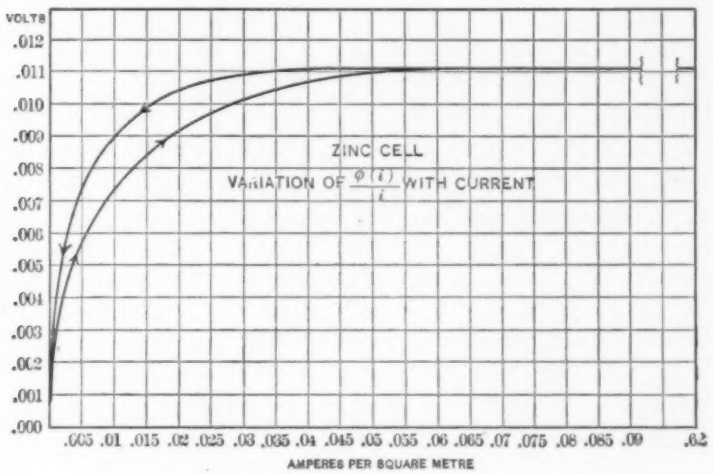

FIG. 4.

through the cells and the current is then increased or diminished a certain amount and held constant at its new value, the e. m. f. $\frac{\varphi(i)}{i}$ varies *directly* with the current.

The first reading of the e. m. f. in these curves was not made at the instant that the circuit was closed on account of having to wait until the galvanometer suspension stopped swinging (this consumed probably three or four seconds), therefore the sudden rise of the e. m. f. from zero, to its initial value plotted, does not appear on the curves.

That there is a difference in the variation of $\frac{\varphi(i)}{i}$ with time when

the cells have stood with no current passing through them and when current has been flowing through them, is clearly shown, in Figs. 1, 2 and 3, by comparing curves I with curves II, III, IV and V. This entirely unexpected result may be explained as follows:

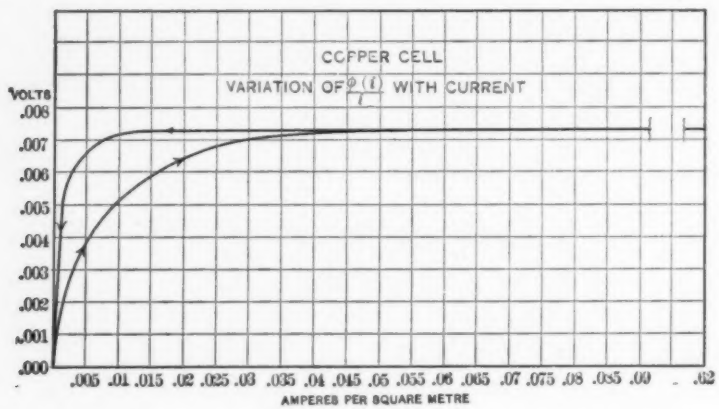

FIG. 5.

After the cells have stood with no current passing through them, the surface of the electrodes and the crystals of the deposited ions may not be as clean as when the current has been established and the crystals are growing. Now the ionic tension or osmotic pres-

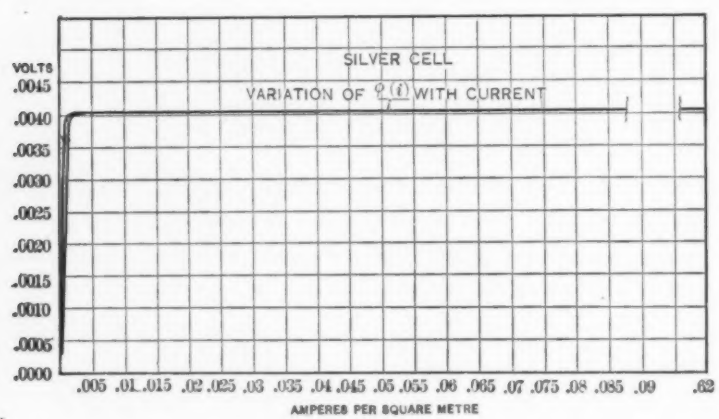

FIG. 6.

sure is probably not so great when the crystals are clean and growing, hence, if so, the initial e. m. f. would not have to be so great to push the current through the cell as when the crystals are dull.

The results as shown by the current e. m. f. curves, Figs. 4, 5 and 6, indicate that the e. m. f. $\frac{\varphi(i)}{i}$ does *not* become zero with the smallest current density used and that, therefore, the expenditure of power $\varphi(i)$ in the cells is due to an irreversible thermo-dynamic action and the laws of thermo-dynamics are not applicable to the action of polarization in an electrolytic cell.

PHYSICAL LABORATORY, LEHIGH UNIVERSITY, June, 1898.

REMARKS ON THE ABOVE PAPER.

It is a question whether the polarization e. m. f. of an electrolytic cell approaches zero with current density on the electrode surface. If it does not, then, even the slowest deposition of ions at an electrode would depend upon a finite supersaturation of the ions in the solution at the electrode; and the electrolytic cell instead of presenting a genuine case of thermo-dynamic equilibrium would present a case of what Duhem calls false equilibrium, the analytical treatment of which is very different from the treatment of true equilibria. The trend of Mr. McNutt's curves for very small current densities does not indicate with certainty that the polarization e. m. f. is other than zero for *zero current.*

The behavior of an electrolytic cell which has been for a long time an open circuit is interesting. When current is started through such a cell the polarization e. m. f. very quickly reaches an abnormally large value and then falls off for some hours. On the other hand, if the cell has been on open circuit for a few moments only the polarization e. m. f. rises steadily when the current is started and reaches its maximum value after several hours.

W. S. FRANKLIN.

A METHOD OF MEASURING THE FREQUENCY OF ALTERNATING CURRENTS.

BY CARL KINSLEY.

ALL users of alternating currents have experienced the difficulty of determining their frequency. A new method by which this important determination can be made quickly and with an error of less than ¼ of 1% will, I think, be of considerable interest.

All methods that have been used include some form of secondary apparatus moving synchronously. A motor can be used[1] but the accurate determination of its velocity, if changing, is of considerable difficulty.

Dr. S. P. Thompson[2] has found the frequency by noting the number of beats due to a rated fork and a telephone driven by the alternating current. The range covered by a single forki s unfortunately much smaller than the variations in the frequency of an alternating current.

An air column vibrating in unison with a telephone diaphragm which is caused to move by the alternating current may be used.[3] This method has the advantage of not requiring a preliminary calibration, but its accuracy depends on the skill of the operator. The error may be kept within ½ of 1%. Professors Ayrton and Perry[4] used a wire; and following them a strip of iron was employed as the synchronous vibrator,[5] the periodicity being changed by varying its length. In this way determinations cannot be made

[1] Trans. Academy of Science of St. Louis, Vol. VII., No. 7, p. 128. I have developed a method by which instantaneous values can be obtained by means of a direct connected series dynamo.

[2] Proc. Phys. Soc., Vol. 14, p. 270.

[3] Kinsley, Phil. Mag., April, 1898.

[3] "Electrician," London, July 5, 1889.

[4] Campbell, Phil. Mag., August, 1896.

quickly, as changes of length must be made with great care. The difficulty of clamping the bar and determining its length will be evident, since in the case of a bar used to determine a frequency of about 16,000 alternations per min., a change of length of 0.007 cm. gave a change of ¼ of 1% in its frequency.

I wish to propose the use of a vibrating strip of iron whose length is not changed. The necessary adjustment of the periodicity of the strip is made by adding a non-magnetic rider to the bar. The loaded bar is made to vibrate by an electro-magnet which is energized by the alternating current.[1] This method is not subject to the serious difficulties of the preceding one and is capable of more rapid and accurate use.

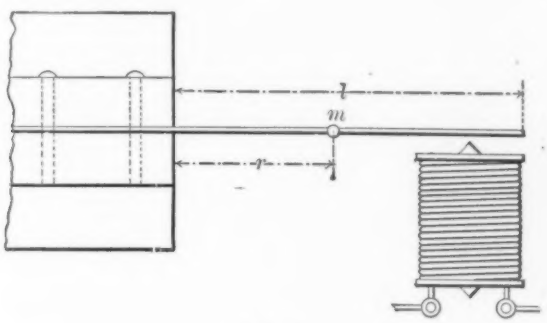

FIG. 1.

For the lateral vibration of a bar loaded at the end Lord Rayleigh proposes the following equation which is obtained by means of a series of approximations.[2]

$$(1) \qquad \frac{1}{p^2} = \frac{l^3}{3\,qk^2\omega}\left(m + \frac{33}{140}\rho\omega l\right).$$

p = number of complete periods per sec.

l = length of the bar in cm.

q = Young's Modulus. (This should be the modulus of elasticity of bending).

[1] After this paper had been written I found that Professors Ayrton and Perry had incidentally mentioned the possibility of using a "magnetic tongue whose natural rate of vibration is varied by altering its moment of inertia." Apparently that method was not further considered by them.

[2] "Theory of Sound," 2d ed., Vol. I., p. 289.

k = radius of gyration of a section of the bar about a line through the axis and perpendicular to the plane of bending.

ω = cross section of the bar in sq. cm.

m = mass of the rider in gms.

ρ = density of the material of the bar.

Also let

h = width of the bar in cm.

τ = thickness of the bar in cm. $\tau^2 = 12k^2$.

t = time in sec. of one complete period.

If the rider is not at the end of the bar a further approximation, which, however, will not increase the error, gives the equation

$$\frac{1}{p^2} = \frac{1}{3qk^2\omega} mr^3 + \frac{33}{140} \frac{\rho}{3qk^2} l^4 \tag{2}$$

where r = distance of the rider in cm. from the supported end of the bar.

We can collect the terms that will be constant when any particular rod is used and then the equation becomes

$$t^2 = Fmr^3 + K, \tag{3}$$

or if a single rider is used the equation may be written

$$t^2 = Cr^3 + K. \tag{4}$$

F, K and C, are then constants depending on the material and dimensions of the rod.

Equations (3) and (4) were found by experiment to be correct. The approximations may have affected the value of the constants, but for our purpose the equations are correct to within 1/10 of 1% when there are widely varying values of t, l, r, m and τ.

I used experimentally one bar of tool steel, one strip of rolled sheet steel and three sizes of band iron. I used two different lengths of the tool steel. With one of the lengths t and r were the only variables. With the other length r and m were varied independently and t was measured. Each series showed conclusively that with this bar F and K of equation (3) are truly constant throughout the complete range possible to employ.

This was no less authoritatively shown by the experiment with rolled sheet steel and the three different sizes of band iron.

In each case the length of the bar was kept constant and only one rider was used, but r was varied from 0 to l. Each rod was used at a different length, and the thickness, varying from 0.0285 to 0.115 cm., was different in each case. The values of the measured constants were substituted in equation (2) and the modulus of elasticity was thus found. When it was noticed that the moduli obtained from the different bars were substantially the same and that the modulus found from each portion of the right hand member of the equation was the same within the limits of experimental error then it was thought that the correctness of equation (2) was sufficiently established for our purpose.

While in the case of different specimens of iron the constants are always nearly the same, for steels they may vary widely. A difference of 40% has been sometimes found. Therefore, if steel is used and the constant assumed, the error might be large.

In order, then, to plot a curve giving the relation between t and r, it is merely necessary to determine two simultaneous values of t and r and from the two equations to then solve for C and K. This calibration can be made with a resonating air column as previously explained or by any other more convenient method. I have generally used a small generator with a gearing arranged to close a chronograph circuit each 100 revolutions. The absolute accuracy of the frequency determinations will depend largely on the determination of these constants.

If relative values are of the most importance and a constant error of a few per cent. is allowable, then it is possible to use a kind of iron whose constants have been determined.

For iron bars equation (2) can be written:

$$t^2 = A\frac{mr^3}{h\tau^3} + B\frac{l^4}{\tau^2}. \tag{5}$$

In general it can be said that there is always a definite ratio between the constants A and B of equation (5) which is independent of the modulus of elasticity. So with iron of the same density it is really

necessary to make only one actual determination of the time of oscillation of the bar and the frequency of the current in order to determine the constants C and K of equation (4).

If, for instance, it is desired to determine the frequency of an alternating current that never gets higher than 64.7 periods per sec., $t = \frac{1}{64.7} = 0.01546$.

The constants found for band iron in the experimental work mentioned above are

$$A = 1.17 \times 10^{-10}, \quad B = 2.13 \times 10^{-10}, \quad \frac{A}{B} = 0.549.$$

Let us assume first that these constants are correct for the kind of iron we employ as a resonator. Find then from the equation the approximate length that will give us resonance when no rider is used. By a slight adjustment the exact length for the condition of resonance will be readily found. Then with the correct value of t and l find the exact value of B for the iron used.

Let us take a rod having the physical constants

$$\tau = 0.056 \text{ cm.} \quad h = 1 \text{ cm.} \quad m = 0.27 \text{ gm.}$$

The approximate value of l when $r = 0$ and t and B are as above is

$$l = \sqrt[4]{\frac{t^2 \tau^2}{B}} = 7.7 \text{ cm.}$$

Since the value of B assumed in this case is the correct value—it will never be far different—we find by experiment that we have the bar thrown into resonance when $t = 0.01546$

$$\therefore B = \frac{t^2 \tau^2}{l^4} = 2.13 \times 10^{-10}.$$

The ratio between A and B given above can now be used to find A, and by substituting the measured values of the constants equation (4) reduces to

$$t^2 = 1.80 \times 10^{-7} r^3 + 2.39 \times 10^{-4};$$

since $$K = B\frac{\rho^4}{\tau^2} = 2.39 \times 10^{-4};$$

$$C = A\frac{m}{h\tau^3} = \frac{0.549\,Km}{h\,\tau\,l^4} = 1.80 \times 10^{-7}.$$

It will be possible, therefore, with this rider to measure frequencies between 64.7 and 56.1 periods per sec. The range can be varied at will by changing the mass of the rider.

This same rod but made somewhat shorter, $l = 5.35$, was at another time used to measure the frequency of the University lighting current which was about 133 periods per sec. It was found easily possible to arrange the rod so that a chance of 1 cm. in the position of the rider showed a change of frequency of only $\frac{1}{4}$ of 1%, while if the rod itself were lengthened, only 0.007 cm. there would be the same change of frequency.

I have found that thin rods, on account of their bending readily, are the most easily used in finding the position of resonance.

It will, I think, be evident from the preceding, that the loaded rod of constant length makes an exceedingly simple, easy and accurate method of determining the frequency of alternating currents.

PHYSICAL LABORATORY OF JOHNS HOPKINS UNIVERSITY, Baltimore, Md.

March 1, 1899.

RADIATION PHENOMENA OF THE BALMAIN LUMINOUS PAINT.

By Leopold Kann.

IN the hope of finding, if possible, a process of picturing very short Hertzian waves which I had succeeded in producing in lengths of from one to two millimeters, I experimented, while in the Physical Laboratory at Vienna, with the fluorescence of the Balmain luminous paint. I had chiefly in mind at that time the extinction of the fluorescence caused by a certain kind of light by another of longer wave-length. My experiments in this direction were not successful, but I succeeded by throwing the spectrum upon a fluorescent screen of this material and then placing the latter upon a photographic plate, in obtaining pictures that showed very well the lines in the red part of the spectrum.

In January, 1899, I again took up these experiments. The intense violet light from my fluorescent screens induced me to inquire more closely into the photographic effectiveness of such light, and I found in working with the Balmain paint some very curious radiation phenomena (late in January). Although my researches are not completed as yet—they having been temporarily interrupted—I wish to publish here a short preliminary report on these phenomena.

Upon a frame of cardboard I mounted strips of different materials which were crossed by other strips. This frame was then placed on a photographic plate in a light-tight box and over it was laid the screen which for some time (one-half hour to six hours) had been exposed to the direct sunlight. This plate was left in the box for different spaces of time (from one-half minute to a few days). It was found that while the light very easily passed through several layers of white paper it did not pass nearly so easily through black paper. It was, indeed, possible to print on the photographic plate very plainly within half a minute the name on a vis-

iting card enclosed in a very heavy envelope. Objects placed upon the black paper also showed their contours upon the plate, but only after a long exposure. The different strips referred to above showed different transparencies which, however, I have not yet arranged in any definite succession. Cardboard, wood and aluminum appeared to be especially transparent. Printed letters on a card were sharply photographed even through a sheet of aluminum.

I believe that these effects are due to the radiation of the Balmain paint and not to the vapors of the materials of which the strips are composed, for the same strips were placed upon a photographic plate and exposed five days without obtaining any blackening of the film. Some of the results, it is true, might be explained by the transparency of some of the strips to the rather intense visible light emitted by the screen, but further experiments taught that screens which had been kept in a dark room for one or two days, although the luminosity ceased after five to six hours, gave essentially the same results. It seems, therefore, that the effects are due to the invisible part of the spectrum, but further experiments are needed to establish this. Besides the repetition and perfecting of the experiments already made, I hope to investigate the influence due to the exciting of the screens by different colors, by X-rays, by different sources of light; also the polarization of this light from the screen, its ability to discharge electrified bodies, etc. The few experiments which I have made in regard to the last named subject gave a negative result. Most of the photographs referred to, although clear, are naturally not very rich in contrast and would lose much in reproduction. I shall, therefore, not give them in the present brief paper.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY, April 2, 1899.

NEW BOOKS.

Theoretical Mechanics. An Introductory Treatise on the Principles of Dynamics. By A. E. H. LOVE. 8vo. Pp. xiv + 379. New York, The Macmillian Co., 1897.

This is a book for the advanced student. After having gone through a good elementary treatise he may enter on its study and not find it too difficult. It will repay him. It is a presentation of the subject, modern in method and marked by clearness and precision of statement.

The book is divided into three parts. The first is preliminary and devoted to kinematics,the key-note being the relativity of motion. The second part is devoted to dynamics, the central idea being that of mass as defined by Newton. The third part consists in the application of the general principles already laid down to what are commonly known as Particle Dynamics and Rigid Dynamics.

The book is very abundantly supplied with examples and problems, there being several hundred of these of all degress of difficulty. In most cases the result is included in the statement of the problem itself. Hardly any of the problems are numerical, almost all being expressed in quite abstract terms.

The book is an attempt to bring the doctrines of Newton into line with the modern doctrine of the relativity of motion. That the ordinary presentation of the fundamental principles is not free from difficulty is shown by the recent work of Professor Holman on "Matter, Energy, Force and Motion" in which the fundamental definitions of the science are re-stated and some changes of terminology suggested. Mr. Love is not quite so radical. In a short "critical note" appended to Chapter VIII, he discusses the foundations of mechanics, but in a rather condensed and scrappy way. It is to be regretted that matter of this kind should be put in small type as if of minor consequence. It is in our opinion one of the very important things in an advanced treatise. The student ought to be informed of the actual state of affairs, of differences of view and of the difficulties underlying the subject. A few references to other writings when special points are discussed would here be of value. If room were needed the author could have omitted say 500 problems, and had enough and to spare remaining.

The book closes with a very meagre account of mechanical units. Mr.

Love follows the ordinary system of the English physicist. In his preface he refers to Mach as one of his guides, but he gives the old definition of mass as "quantity of matter," without a hint that Mach utterly repudiates this so-called definition.

This is no objection to the term mass if its use is confined to the laboratory. But to speak of the "mass of a railway carriage" (p. 335) when people in general say "weight," is objectionable. The absurd fad of having a system of names for the lecture room different from that of the workshop or of the counting house was in great favor a short time ago. Common terms were used in a special sense, and new terms introduced. Attention was first called to this abuse by Professor Greenhill. Largely owing to his persistent efforts it is dying out and we hope soon to see the last of it.

As an evidence of its decay we fail to find in Mr. Love's book the lately popular monstrosity, the "poundal." As a further evidence Professor Holman in his work referred to above says (p. 153): "Notwithstanding the prevalence of the use of the term *mass* among physicists in recent years, conservative opinion, notably among eminent chemists, is opposed to the employment of that term in place of the time-honored "weight" to denote quantity of substance or matter as measured by the process of "weighing." Engineers have held this view all along.

We hope Mr. Love in his next edition will be a little less didactic and give more space to the consideration of different views. Let us have more "critical notes." The student will gain, even if he does not solve so many problems.

T. W. WRIGHT.

SCHENECTADY, March 10.

Traité élémentaire de Mecanique Chimique. Par P. DUHEM. Tome III. Pp. 380. Paris, Hermann, 1898.

This concluding volume of Duhem's Treatise is devoted to solutions and to homogeneous mixtures. In the second chapter the author gives a very full examination of the question as to what extent non-observable aspects of a system, such for example as the physical state of a body in solution, may be inferred by thermodynamic methods from observed data; and of the question as to whether the validity of any thermodynamic relation can be affected by hypotheses concerning these non-observable aspects of a system.

The great power of thermodynamics pure and simple is due to the fact that all of its developments are based upon sensible things and that all of its results—not depending upon special hypotheses—are such as can be directly verified by measurements of volume, pressure, temperature, density, etc. The results of thermodynamics refer therefore to concrete

values and not to abstract *elements*. It is this real quality of thermodynamics which has led many of our physical chemists to set themselves in speculative opposition to atomistic methods forgetting that very many of the most striking and useful developments of thermodynamics are the results of special hypotheses of an unreal character incapable in themselves of experimental verification.

The exponents of energetics, for they are more in need of the instruction than are those who frankly admit that their scientific method rests in large part on unreal atomistic constructions, would do well to heed the instruction of Hugo Münsterberg, than whom no one is doing more to clear up the philosophy of science. "The naturalist knows, if he understands the philosophical basis of his work, and is not merely a technical craftsman, that natural science means, not simply a cast and copy of the reality, but a special transformation of reality, a conceptual construction of unreal character in the service of special logical purposes."

The present writer feels some disappointment in Duhem's Treatise. The first volume seemed to be very properly analytical like the first part of Maxwell's great work; but on the whole Duhem's work is very meager in "conceptual construction," that "New Engine," helping the mind as tools help the hand, which Bacon recognized as a want in his time. It may be, however, that certain branches of science are to become *algebraic constructions* and Thermodynamics may be one of these as the present writer suggested in his notice of Duhem's first volume on *Mecanique Chimique*. If this be the case, then very certainly the speculative opponents of atomistics cannot be allowed the satisfaction of their claim that this *algebraic construction* is of less unreal character than any *conceptional construction*, however atomic or molecular it may be!

W. S. FRANKLIN.

Der Tägliche Wärmeumsatz im Boden und die Wärmestrahlung zwischen Himmel und Erde. By THEODOR HOMÉN. 147 pp. Leipzig, Wilhelm Engelmann. 1897.

In this volume Dr. Homén of the University at Helsingfors presents the results of a very interesting research upon solar radiation and the daily interchange of heat between the earth's crust and the atmosphere. While the results obtained are perhaps chiefly of importance to the meteorologist and the geologist, the methods pursued cannot fail to interest physicists in the highest degree.

Dr. Homén's experiments, which were extensive, running through several summer seasons, included daily measurements and, for periods of many days, hourly measurements of the temperature of the upper layers

of the earth's crust. Readings were taken at ten different depths beneath the surface, varying from one to seventy centimeters, in sandy soil, in clay soil, beneath the vegetation of moorland and meadows, in granite rock, and in the waters of a small lake. Corresponding measurements were taken in the air at heights above the surface running to ten meters. Determinations of the formation of dew with estimates of the amount of heat thus set free were likewise carried on, and measurements were made of the direct radiation from the sun, of that from the open heavens and of the return radiation from the earth's surface. Very complete data were thus obtained for the discussion of the complicated problem with which the research deals. The measurements of soil temperatures were made by means of special thermometers inserted with every possible precaution to the desired depths beneath the surface of the ground. For the measurements on radiation a modification of Ångstrom's excellent instrument of electrically compensated metal strips[1] was employed.

The memoir is accompanied with several curve sheets plotted to a large scale upon which the results of the inumerable thermometric determinations in various soils are graphically shown. Some of these, as for example the diagram showing the daily fluctuations of temperature beneath the surface of granite rock, are of extraordinary interest.

E. L. N.

The Barometrical Determination of Heights. A Practical Method of Barometrical Levelling and Hypsometry for Surveyors and Mountain Climbers. By F. J. B. CORDEIRO. 28 pp. New York, Spon & Chamberlain. 1898.

The form given to this little volume is that of a thin pocket book with flexible covers. It contains the standard formulæ for the determination of heights by means of the mercury barometer, together with many details concerning the handling of that instrument which will be of use to explorers and all who have occasion to make such measurements. The author condemns the use of the aneroid barometer altogether, basing his opinion in part upon Whymper's description of the irregularities of that instrument. While for purposes of *high precision* a mercurial barometer is doubtless essential, it is only fair to state that a well constructed aneroid barometer with proper temperature corrections will often be found useful in the *approximate* determination of heights. The reviewer has carried such a barometer to many elevated points in Switzerland and elsewhere and has found that in settled weather, by setting the scale of heights before leaving a position of known elevation above the sea, that the reading of the instrument agreed in almost all instances very well indeed with the determined heights of other positions to which it might be

[1] Ångstrom. See PHYSICAL REVIEW, Vol. I., p. 365.

carried in the course of the same day. In Switzerland where the height of almost any accessible point has already been determined with considerable precision one can check up the readings of such an instrument very conveniently. In scores of instances the agreement of the indications of the aneroid with the established heights was a surprise to one who knew the sources of error to which such instruments are necessarily subject.

The confusion in the use of units of measure displayed in this book affords a striking illustration of the chaotic condition to which we have come in our gradual and painful transition from the British to the metric system. That any writer should deem it advantageous to convert the weight of a liter of air in grams to grains Troy is at the present day well nigh inconceivable. That within the same table the values of gravity should be given at Paris (of all places) in feet and in the same column the values for other localities in centimeters, is another instance of the lamentable confusion into which many writers of these days have fallen. For expressing gravity in dynes Dr. Cordeiro can find high authority, especially in Washington, but it is a questionable practice to depart from the established definition which measures gravity in terms of acceleration and not of force. The needless annoyance to which readers are put by the confusion of systems of measurements, to which reference has already been made, is well illustrated on page 14, where in the course of a single computation we have to deal with the weight of a liter of mercury in grams, temperatures in centigrade degrees, pressures in inches of mercury, the weight of a cubic foot of air. Immediately after this lapse from the metric system into British measures the formula demands us to consider the coefficient of expansion per degree centigrade, and when we come to the correction for moisture we are referred to a table in which the pressures of aqueous vapor are given in millimeters of mercury.

E. L. N.

Physical Geography. By WILLIAM MORRIS DAVIS, assisted by WILLIAM HENRY SNYDER. Pp. xvii + 428. Boston, Ginn & Company. 1898. (Received).